UNIVERSITY OF LIVERPOOL

# Metastable Vacua and Geometrical Engineering

Thesis submitted in accordance with the requirements of the University of Liverpool for the degree of Doctor in Philosophy by Ben John Wetenhall

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*Department of Mathematical Sciences*
*University of Liverpool*

*For my Mum*

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# Abstract

In this thesis we work on the geometrical engineering of metastable supersymmetry breaking models in type II string theory. First we work out the T-duals of the brane configurations of the ISS model and of the metastable model with wrapped branes and anti-branes, and then lift the latter to M theory. Secondly, we use normalisable and non-normalisable deformations to describe stringy realisations of metastable vacua in $\mathcal{N} = 1$, $SU(N)$ gauge theories and we enlarge the class of metastable vacua to include branes and anti-branes wrapped on cycles of deformed $A_n$ singularities. Lastly, we perform a T-duality on the brane configuration of the metastable model with a general potential for the adjoint field in order to obtain a geometrical picture.

# Contents

# Chapter 1

# Introduction

Dualities are very powerful tools in string theory. Having an alternate description of a phenomenon can shed light on many different aspects of it. In one setup certain features may be more visible than in others and having complementary pictures can reveal new features of the theory.

Much of the impetus for the recent developments in string theory has come from new discoveries in field theory. During the nineties Seiberg led an effort to exploit the special simplifications of $\mathcal{N} = 1$ supersymmetric field theories to discover the behaviour in the strong coupling region. He discovered that in many cases there exists a remarkable nontrivial dual description –so called Electric-Magnetic duality– relating a region with weak coupling to a region with strong coupling [1, 2]. This of course is a powerful duality as it allows the use of perturbation theory in the weakly coupled theory to study effects in the strongly coupled theory.

In chapter 2 we introduce some of the basic field theory tools we will use. We give a lightning review of some aspects of $\mathcal{N} = 2$ and $\mathcal{N} = 1$ supersymmetric gauge theories, in particular we show how supersymmetry may be broken spontaneously by non-zero expectation values for some of the components of the auxiliary fields. We introduce $\mathcal{N} = 1$ SQCD and we show how Seiberg's duality appears in the context of SQCD.

The low energy theory of a IIA brane configuration of fourbranes and fivebranes can give $\mathcal{N} = 2$ and $\mathcal{N} = 1$ supersymmetric gauge theories [3]. We will see at the end of Chapter 4 that the IIA brane configurations are T-dual to geometrical setups in IIB string theory with branes wrapping certain cycles of a Calabi-Yau manifold [4–7]. The low energy limit of the two approaches gives the same effective field theory and features of these gauge theories can be uncovered in string theory. Lifting the IIA brane configurations to the strong coupling limit of IIA string theory – M theory– gives us another playground in which to test the models [8, 9].

The IIA string theory brane description of Seiberg's duality was first shown in [10, 11] where the electric and magnetic theories are obtained from one another through an exchange of the positions of the fivebranes. In IIB string theory, Seiberg's duality has been shown to be a toric duality [12, 13]; the toric duality implies a flop of one of the cycles of the Calabi-Yau manifold.

One current major problem both in field theory and string theory is how supersymmetry (SUSY) is broken and how to realise this breaking in string theory. There are many ways to break supersymmetry but it is not known which way is correct. One such way is dynamical SUSY breaking (DSB), an elegant method which preserves naturalness. However, it is hard to achieve in model building mainly because all the lowest energy states have to be SUSY breaking. In particular the Witten index predicts a theory must have a certain number of supersymmetric groundstates [14] and it can be very difficult to circumvent this.

Recently the idea of metastable vacua with a lifetime much longer than the universe in simple theories such as massive SQCD has been put forward by making use of Seiberg's duality [15]. It abandons the idea that models of DSB must have no SUSY vacua and is much simpler and easier to implement in realistic models.

Chapter 3 covers the field theory of some models of metastable supersymmetry breaking; we also discuss quantum corrections to the models and their stability. The ISS model is briefly reviewed along with some generalisations: the model of Girardello *et al* with superpotential for the adjoint field [16], the model of Giveon and Kutasov with gauge singlets (an extra adjoint field for flavour group) in the superpotential [17] and we discuss the field theory of the metastable model of Vafa *et al* [18].

The ISS model has been embedded in string theory in [19–21], the model of Girardello *et al* in [22] and the model of Giveon and Kutasov in [23]. The fact that the configurations are non-supersymmetric is due to a misalignment of fourbranes. The IIA brane configurations are lifted to a single M5 brane. In [21] it was pointed out that in the brane configurations of the ISS model there is an inconsistency when lifting to M theory due to an inconsistency of boundary conditions at infinity for the bending of the fivebranes.

Chapter 4 shows how we can get $\mathcal{N} = 2$ and $\mathcal{N} = 1$ effective field theories, similar to the ones discussed in Chapters 2 and 3, from configurations of branes in IIA string theory. We show how to realise the electric and magnetic pictures of $\mathcal{N} = 1$ SQCD from brane configurations, including how to interpolate between the two configurations thus realising Seiberg's duality. In the final part of Chapter 4 we show how to construct the ISS and metastable Giveon and Kutasov models in IIA brane configurations and how to use T duality to interpolate between brane configurations in IIA and geometrical setups in IIB.

In Chapter 5 we show how $\mathcal{N} = 2$ and $\mathcal{N} = 1$ effective field theories arise in geometrical configurations in IIB string theory. We present the duality of [24] where at large $N$ a geometric transition occurs where a resolved conifold singularity of a Calabi-Yau manifold is replaced by a deformed conifold singularity. We also review the geometric transition in the case of a general superpotential for the adjoint field first considered in [25] and how we can take a T duality to go from a geometrically engineered IIB configuration to a IIA brane configuration [26]. At the end of Chapter 5, we review the model of [18] with branes and anti branes wrapped on rigid $\mathbf{P}^1$ cycles in the same homology class. The model has the usual geometric transitions and the strongly coupled theories contain positive and negative flux.

Chapter 6 reviews the works of [8, 9]. We show that the M theory lift of IIA brane configurations with four and fivebranes is to a single M theory fivebrane. We also show the effect of a geometric transition of the type studied in Chapter 5 in M theory [4–6].

In [27] they showed how to get around the inconsistency in the M theory picture of the ISS model. The changes in the asymptotic conditions of the fivebranes were considered to have take place after an infinite time. The Seiberg dual of their brane configuration contains D4 branes and anti D4 branes with the ends of the branes on the same NS brane but different NS′ branes. Tachyon condensation of the branes and anti branes on the NS brane gives bending of the fourbranes. A backreaction reaches the NS′ branes after an infinite time.

In Chapter 7, which is based on the publication [28] done in collaboration with Radu Tatar, we study the metastable vacua of [15] and [18]. The geometrically engineered configuration of [18] is translated into a IIA brane configuration and the corresponding lift to M theory, which is similar to [4–6], is performed. The M5 brane will have several disjoint parts and each levels out into planar M5 branes which reduce to disjoint deformed conifold singularities. The D4 branes are mapped into D5 branes and the anti-D4-branes are mapped into anti-D5-branes. The annihilation between branes and anti-branes is prevented by separation of the D4 and anti-D4 branes on intervals of NS branes.

We also present the geometrical picture dual to the IIA brane configurations of [19–21]. The geometry of the case with massless flavours or with the flavours having been integrated out is much simpler [4–7]. When the mass of the flavours is smaller than the scale of the theory the situation is more complicated. We first consider the massless case and then deform the electric and magnetic pictures by adding masses or vevs for the flavours.

We start with a resolved conifold singularity and wrap $N_c$ colour D5-branes on the corresponding compact $\mathbf{P}^1$ cycle and $N_f$ flavour D5-branes on the non-compact holomorphic 2-cycle. We make a non-holomorphic deformation of the $\mathbf{P}^1$ cycle so that it touches the non-compact 2-cycle; the result is a holomorphic cycle in a different complex structure. We can now do the usual Seiberg duality of [10,11] as a flop in the geometry. The complex structure remains unchanged by the flop and the flavour and colour branes remain aligned. We then deform back the $\mathbf{P}^1$ cycle to the original complex structure, the non-compact flavour 2-cycle remains unchanged but it is no longer holomorphic in the original complex structure. The tachyon mode after the rotation of the cycles implies a recombination of some of the cycles. The

$\mathbf{P}^1$ cycle changes into a non-compact holomorphic cycle and a non-compact non-holomorphic cycle.

In Chapter 8, based on the publication [29] also in collaboration with Radu Tatar, we present a general recipe for dealing with models with fivebranes wrapped on $\mathbf{P}^1$ cycles of deformed ADE singularities. The T-dual configurations contain D4-branes, anti-D4-branes and NS5 branes. The corresponding tachyon condensation gives a misalignment of fourbranes signalling the breaking of supersymmetry.

We start with a resolved $A_n$ singularity and wrap $N_i$ D5-branes on each of the $n$ $\mathbf{P}^1$ cycles obtaining an $\mathcal{N}=2$, $\prod_{i=1}^{n} SU(N_i)$ gauge theory with bifundamental matter between neighbouring groups. To break supersymmetry to $\mathcal{N}=1$ we add a quadratic superpotential for each of the adjoint fields $\Phi_i$ of the $SU(N_i)$ factors. To smoothen out the geometry $n^2$ deformations are needed [6]. Out of these $n(n+1)/2$ of the deformations are normalisable deformations and $n(n-1)/2$ are non-normalisable deformations. The normalisable deformations correspond to $\mathbf{P}^1$ cycles which can go through the usual geometric transition to $S^3$ cycles, the non-normalisable deformations measure the distance between the $\mathbf{P}^1$ cycles corresponding to masses or vevs for the flavours in the field theory.

A Seiberg duality on the system is a flop in the geometry. The change in intersection number of the $\mathbf{P}^1$ cycles is related to a change in complex structure. Tachyon condensation between the D5-branes and anti-D5-branes determines a closure of some of the non-normalisable deformations and a recombination of some normalisable and non-normalisable cycles. The cycles that have been combined with some of the non-normalisable cycles are non-holomorphic in the original complex structure and this is related to the metastability of the system.

Chapter 9, is based on the publication [30] again in collaboration with Radu Tatar. In this chapter we study the brane configuration of [23] whose field theory was presented in [17]. We show how to translate the brane configuration into a system with four and fivebranes only and we perform a T-duality on this configuration to obtain a geometrical picture.

We start with an $\mathcal{N}=2$, $SU(N_c)\times SU(N_f)$ theory and break supersymmetry down to $\mathcal{N} = 1$ by adding a mass for the field in the adjoint representation of $SU(N_c)$, such that there are vevs for the electric quarks and the gauge group is broken to $SU(N_f - k) \times SU(N_c - k) \times SU(k)$, where $0 < k < \max\{N_f, N_c\}$.

A Seiberg duality is a flop in the geometry and the following tachyon condensations determine the ranks of the gauge groups and a reorientation of some of the cycles. Some cycles become non-holomorphic in the complex structure; this is related to the metastability of the model. A deformation of the non-holomorphic cycle to a holomorphic cycle gives a supersymmetric theory and this deformation is related to the lifetime of the metastable vacua.

We end the thesis with some conclusions and possible future directions.

# Chapter 2

# Supersymmetry

In this chapter we will introduce some basics of supersymmetric gauge theories focusing briefly on the cases of $\mathcal{N}=2$ but mainly on the case of $\mathcal{N}=1$. The $\mathcal{N}=2$ gauge theories may be broken to $\mathcal{N}=1$ by adding a mass for the adjoint field in the vector multiplet. We move on to the supersymmetric version of Quantum Chromodynamics: Supersymmetric QCD or SQCD. We will see it exhibits a powerful strong/weak coupling duality: Seiberg Duality.

We will assume familiarity with the basics of supersymmetry and the superfield formalism. For reviews on the topics covered in this chapter see [31–37].

## 2.1 Supersymmetric Field Theories

### 2.1.1 $\mathcal{N}=2$ Supersymmetric Field Theories

First we consider $\mathcal{N}=2$ supersymmetric gauge theories with gauge group $G$ which have two kinds of matter multiplets: vector multiplets and hypermultiplets. The vectormultiplet contains a gauge field $A_\mu$, two Weyl fermions $\lambda_\alpha$, $\psi_\alpha$ and a complex scalar $\phi$, all transforming in the adjoint representation of $G$. In terms of $\mathcal{N}=1$ supersymmetry the vectormultiplet decomposes into a vector superfield

$$V=-\theta\sigma^\mu\bar{\theta}A_\mu-i\bar{\theta}^2(\theta\lambda)+i\theta^2(\bar{\theta}\bar{\lambda})+\frac{1}{2}\theta^2\bar{\theta}^2D\ , \tag{2.1}$$

with supersymmetric field strength

$$\mathcal{W}_\alpha = \bar{\mathcal{D}}^2 \left(e^{2V}\mathcal{D}_\alpha e^{-2V}\right) , \tag{2.2}$$

where

$$\mathcal{D}_\alpha = \partial_\alpha + i\sigma^m_{\alpha\dot{\beta}}\bar{\theta}^{\dot{\beta}}\partial_m ; \tag{2.3}$$

and a chiral superfield

$$\Phi = \phi + \sqrt{2}\theta\psi + \theta^2 F , \tag{2.4}$$

where $\theta$ and $\overline{\theta}$ are the superspace coordinates, $\sigma$ are the Pauli matrices, $\mathcal{D}$ is the covariant derivative and $D$ and $F$ are auxiliary fields. The low energy Lagrangian describing the vectormultiplet is

$$\mathcal{L}_{vec} = \text{Im Tr}\left[\tau\left(\int d^4\theta\Phi^\dagger e^{-2V}\Phi + \int d^2\theta\mathcal{W}_\alpha\mathcal{W}^\alpha\right)\right] , \tag{2.5}$$

where the trace runs over the group $G$, $\tau$ is given by

$$\tau = \frac{\theta}{2\pi} + \frac{i}{g^2_{SYM}} , \tag{2.6}$$

and $g_{SYM}$ is the coupling of the supersymmetric Yang-Mills theory.

The hypermultiplets consists of, again in terms of $\mathcal{N} = 1$ SUSY, two chiral superfields, $Q$, $\tilde{Q}$ in a representation $R$ of the gauge group. The low energy Lagrangian describing the hypermultiplet is

$$\mathcal{L}_{hyper} = \int d^4\theta\left(Q^\dagger e^{-2V}Q + \tilde{Q}^\dagger e^{-2V}\tilde{Q}\right) + \int d^2\theta\tilde{Q}\Phi Q + \text{c.c.} , \tag{2.7}$$

where $V = V_a T^a$ $(a = 1, \cdots, \dim G)$, and $T^a$ are generators of $G$ in the representation $R$.

The general form of the Lagrangian consistent with $\mathcal{N} = 2$ SUSY is

$$\mathcal{L}_{vec} = \text{ImTr}\left[\int d^4\theta\frac{\partial\mathcal{F}(\Phi)}{\partial\Phi_i}e^{2V}\bar{\Phi}_i + \frac{1}{2}\int d^2\theta\frac{\partial^2\mathcal{F}(\Phi)}{\partial\Phi_i\partial\Phi_j}\mathcal{W}^i_\alpha\mathcal{W}^\alpha_j\right] . \tag{2.8}$$

Here $\mathcal{F}(\Phi)$ is a holomorphic functional known as the prepotential. It determines the low energy $U(1)_R$ gauge coupling matrix $\tau_{ij}$ as

$$\tau_{ij} = \frac{\partial^2 \mathcal{F}}{\partial\phi_i \partial\phi_j} \,, \tag{2.9}$$

and the metric on the moduli space

$$ds^2 = \mathrm{Im}\, \tau_{ij}\; d\phi_i d\bar{\phi}_j \,. \tag{2.10}$$

We now turn to $\mathcal{N} = 1$ gauge theories which will be the main subject of this thesis.

### 2.1.2 $\mathcal{N} = 1$ Supersymmetric Field Theories

$\mathcal{N} = 1$ supersymmetric field theories can be constructed with chiral superfields $\Phi^i$, antichiral superfields $\Phi^{\dagger i}$, and vector superfields $V^a$ transforming in the adjoint representation of the gauge group. The most general Lagrangian for the $\Phi^i$ and $V^a$ with at most two derivatives takes the form

$$\begin{aligned} \mathcal{L} \;=\; & \int d^4\theta\, K(\Phi^\dagger, e^{V\cdot t}\Phi) + \left(\frac{-i}{16\pi}\right) \int d^2\theta\, \tau(\Phi)\, \mathcal{W}^{\alpha a}\mathcal{W}_{\alpha a} + \text{h.c.} \\ & + \int d^2\theta\, W(\Phi) + \text{h.c.}\,, \end{aligned} \tag{2.11}$$

where $\tau$ is the combination of the gauge coupling and the $\theta$ parameter,

$$\tau = \left(\frac{\theta}{2\pi} + \frac{4\pi i}{g^2}\right) \,. \tag{2.12}$$

We now explain the rest of the terms featuring in the Lagrangian (2.11).

The first term of this Lagrangian is a non-linear sigma model for the chiral superfields $\Phi^i$. The metric $g^{ij}$ for this is derived from the Kähler potential $K(\Phi, \Phi^\dagger)$ a vector superfield:

$$g_{ij} = \frac{\partial^2 K}{\partial\Phi^{\dagger i}\partial\Phi^j} \,. \tag{2.13}$$

The third term of the Lagrangian contains the superpotential $W(\Phi)$

a holomorphic functional of $\Phi$, i.e. a functional only of $\Phi$. Only non-perturbative corrections can modify the superpotential [38] and these may lift some or all of the classical moduli space. Since $W$ is a holomorphic function of $\Phi$, in many cases the form of the exact quantum superpotential can be determined exactly from symmetry arguments. Having the exact form for the superpotential can lead to non-trivial results including revealing non-perturbative behaviour. The quantum corrections to the Kähler potential are in general more complicated and are not under control. This is not such a problem as it seems since as long as the Kähler potential is non-singular, the vacuum structure is unaffected. For most examples there is usually some evidence for the corrections to the Kähler potential to be under control as we will see in Section 3.1.

### 2.1.3 Supersymmetric Vacua

The vacuum state of the theory can be found from the Lagrangian (2.11). In supersymmetric theories the energy of any state $\langle H\rangle$ satisfies the condition $\langle H\rangle \geq 0$. A supersymmetric vacuum has zero energy as the state with $\langle H\rangle = 0$ is annihilated by the SUSY generators.

To find the vacuum state of the effective Lagrangian, we minimise the potential energy:

$$V = F_i^\dagger g^{ij} F_j + \frac{1}{2} g^2 (D^a)^2 \ , \qquad (2.14)$$

where $g$ is the coupling constant defined by (2.12), $g^{ij}$ is the metric (2.13), and the auxiliary fields $F_j$ and $D^a$ are given by

$$\begin{aligned} F_j &= \frac{\partial W}{\partial \Phi^j} \ , \\ D^a &= \sum_i \Phi^{\dagger i} t^a \Phi^i \ , \end{aligned} \qquad (2.15)$$

where $t^a$ represents the gauge group generators on $\Phi$. The conditions

$$\langle F_j \rangle \neq 0 \quad \text{or} \quad \langle D^a \rangle \neq 0 \ , \qquad (2.16)$$

signal the breaking of supersymmetry. We call $F_j = 0$ and $D^a = 0$ $F$-flatness and $D$-flatness respectively. If these conditions can be both be satisfied simultaneously then we have a supersymmetric vacuum state.

## 2.2 Supersymmetric Quantum Chromodynamics

Supersymmetric quantum chromodynamics or SQCD is the supersymmetric generalisation of regular QCD. It is an $\mathcal{N} = 1$ supersymmetric, $SU(N_c)$ gauge theory with $N_f$ flavours $Q^i, \tilde{Q}_i, \quad (i = 1, \ldots, N_f)$ in the fundamental and antifundamental representations of the gauge group respectively. In this section we will study classical SQCD and see some of the effects of quantum corrections.

At the classical level this theory has an $R$-symmetry

$$\theta \longrightarrow e^{-i\alpha}\theta \ . \tag{2.17}$$

In the quantum theory this symmetry is however anomalous, the $R$-symmetry can be combined with the anomalous $U(1)$ flavour symmetry to form an anomaly free $R$-symmetry. The full non-anomalous global symmetry of the model is

$$G = SU(N_f) \times SU(N_f) \times U(1)_B \times U(1)_R \ . \tag{2.18}$$

Here $U(1)_B$ is the analog of the usual baryon number and the $SU(N_f) \times SU(N_f)$ factor is the analog of the usual chiral symmetry of QCD. The charges under the global symmetry are shown below

$$\begin{array}{c|c|c|c|c|c} & SU(N_c) & SU(N_f) & SU(N_f) & U(1)_B & U(1)_R \\ \hline Q & \square & \square & \mathbf{1} & 1 & \frac{N_f-N_c}{N_f} \\ \tilde{Q} & \bar{\square} & \mathbf{1} & \bar{\square} & -1 & \frac{N_f-N_c}{N_f} \end{array} \ . \tag{2.19}$$

An $\mathcal{N} = 1$ supersymmetric Yang-Mills theory with gauge group $G$ and chiral superfields in the representations $R_i$ such as SQCD has a one-loop $\beta$

function of the form [39, 40]

$$\beta(g) = -\frac{b_0}{(4\pi)^2} g^3 , \tag{2.20}$$

where

$$b_0 = 3C_2(G) - \sum_i C(R_i) , \tag{2.21}$$

and $C_2(R)\mathbf{1} = (t^a t^a)_R$ is the quadratic Casimir operator with $C(R)\delta^{ab} = \mathrm{tr}_R[t^a t^b]$, and $G$ denoting the adjoint representation. For the case of SQCD, the $\beta$ function coefficient is given by

$$b_0 = 3N_c - N_f . \tag{2.22}$$

### 2.2.1 Classical SQCD

We now explore some of the classical properties of $\mathcal{N} = 1$ SQCD with gauge group $SU(N_c)$.

If we have SQCD with $N_f < N_c$ massless flavours of quarks, the moduli space of vacua is $N_f^2$ dimensional and it can be labelled by the gauge invariant meson fields

$$M^i_j \equiv Q^i \tilde{Q}_j , \qquad i, j = 1, ..., N_f . \tag{2.23}$$

We can see the appearance of the meson as follows. The gauge group can be maximally broken to $SU(N_c - N_f)$. The quarks have $2N_cN_f$ complex components. Following the breaking of the gauge group $N_c^2 - (N_c - N_f)^2$ are eaten by the Higgs mechanism leaving $N_f^2$ massless degrees of freedom- the meson $M^i_j$.

In the range $N_f \geq N_c$ new gauge invariant fields appear, these are $\binom{N_f}{N_c}$ baryon fields defined by

$$\begin{aligned} B^{i_1 i_2 \cdots i_{N_c}} &= \epsilon^{\alpha_1 \alpha_2 \cdots \alpha_{N_c}} Q^{i_1}_{\alpha_1} Q^{i_2}_{\alpha_2} \cdots Q^{i_{N_c}}_{\alpha_{N_c}} , \\ \tilde{B}_{i_1 i_2 \cdots i_{N_c}} &= \epsilon_{\alpha_1 \alpha_2 \cdots \alpha_{N_c}} \tilde{Q}^{\alpha_1}_{i_1} \tilde{Q}^{\alpha_2}_{i_2} \cdots \tilde{Q}^{\alpha_{N_c}}_{i_{N_c}} . \end{aligned} \tag{2.24}$$

In this range of flavours the gauge group can be completely broken by the Higgs mechanism, and the complex dimension of the classical moduli space is $2N_cN_f - (N_c^2 - 1)$. There are many classical constraints relating the baryon and meson fields. To illustrate this consider the case of $N_f = N_c$, the constraint on the system is

$$\det M - B\tilde{B} = 0 \ . \tag{2.25}$$

Accounting for this condition gives the correct dimension of moduli space $N_c^2 + 2 - 1 = N_c^2 + 1$.

The classical moduli space of SQCD is in general very complicated. The set of all such classical constraints is yet to be determined.

### 2.2.2 Quantum SQCD

We now consider how quantum corrections affect the classical picture of SQCD. We saw in the classical case that when $N_f < N_c$ we have an $N_f^2$ dimensional moduli space labelled by the mesons $M^i_j$, with points with enhanced unbroken gauge symmetries. Quantum corrections drastically modify this picture due to the fact that the theory generates a non-perturbative superpotential for $M$. Affleck, Dine and Seiberg [41] found the exact superpotential compatible with all symmetries and holomorphy:

$$W_{ADS} = (N_c - N_f)\left(\frac{\Lambda^{3N_c - N_f}}{\det M}\right)^{\frac{1}{N_c - N_f}} \ , \tag{2.26}$$

where $\Lambda$ is a dynamically generated scale. $W_{ADS}$ may indeed be generated by gaugino condensation in the unbroken gauge group $SU(N_c - N_f)$ for $N_f \leq N_c - 2$, and by instantons for $N_f = N_c - 1$.

Computing the potential, we find that $W_{ADS}$ has no minimum at a finite value of the fields *i.e.* it exhibits runaway behaviour to infinite values of the fields. By adding masses for all the quarks by modifying the superpotential to

$$W \to W_{ADS} - m_i^j Q^i \tilde{Q}_j, \quad i,j = 1, \ldots, N_f \ , \tag{2.27}$$

we can stabilise the runaway behaviour and we have the $N_c$ vacua of pure

$\mathcal{N}=1$, $SU(N_c)$ SYM.

For the case of $N_f = N_c$ $W_{ADS}$ is singular and we find $W_{ADS} = 0$. There are still however quantum effects. In particular the classical constraint (2.25) is modified to

$$\det M - B\tilde{B} = \Lambda^{2N_c} \ , \tag{2.28}$$

and the quantum effects smooth out the classical moduli space.

In the regime $N_f = N_c + 1$ the baryons can be dualised to fields with one flavour index, $B_i = \epsilon_{ii_1\cdots i_{N_c}} B^{i_1\cdots i_{N_c}}$. Classically, the low energy degrees of freedom $M^i_j$, $B_i$, $\tilde{B}_i$ satisfy the constraints:

$$\det M (M^{-1})^j_i - B_i \tilde{B}^j = M^i_j B_i = M^i_j \tilde{B}^j = 0 \ , \tag{2.29}$$

Again quantum effects modify the classical constraint and the mesons and baryons can be thought of as independent fields, with the superpotential

$$W_{eff} = -\frac{\det M - M^i_j B_i \tilde{B}^j}{\Lambda^{2N_c - 1}} \ . \tag{2.30}$$

The equations of motion of the quantum superpotential (2.30) give the classical constraints (2.29).

In the case $N_f > N_c + 1$ there is no known description of the quantum moduli space in terms of a sigma-model for the gauge invariant degrees of freedom $M$, $B$. For $N_f \geq 3N_c$ the theory is not asymptotically free and at low energies the quarks and gluons are free, this theory is referred to as being in a free electric phase, since it has a QED-like potential $V(R) \sim 1/R \log R$.

For $N_f < 3N_c$ the theory is asymptotically free, in particular if $N_f$ is very close to $3N_c$ there is a weakly coupled infrared fixed point that can be studied perturbatively and describes interacting quarks and gluons. Electrically charged sources have a potential $V(R) \sim \alpha^*/R$, and we say that this type of theory is in a non-abelian Coulomb phase.

## 2.3 Seiberg's Duality

Theoretical effort in mid-1990s mainly due to Seiberg in a series of papers [1,2] led to a dramatic break-through in the understanding of strongly coupled $\mathcal{N}=1$ SUSY gauge theories. We now have a detailed understanding of the IR behaviour of many strongly coupled $\mathcal{N}=1$ theories, including the phase structure of such theories. In this section we look at the first example of Seiberg duality which appears in $\mathcal{N}=1$ SQCD.

### 2.3.1 Seiberg Duality for SQCD

Seiberg [1] found a non-trivial duality between two sets of theories that at long distances flow to the same IR fixed point. The duality states that the following two theories are equivalent in the infrared:

1. Electric SQCD, with gauge group $G_e = SU(N_c)$, and $N_f$ fundamental flavours of quarks $Q^i$, $\tilde{Q}_i$.

2. Magnetic SQCD, with gauge group $G_m = SU(N_f - N_c)$, $N_f$ flavours of dual quarks $q_i$, $\tilde{q}^i$, a gauge singlet dual meson chiral superfield $M^i_j$, and superpotential

$$W_{mag} = M^i_j q_i \tilde{q}^j \ . \tag{2.31}$$

We use the terms electric and magnetic in analogy with the duality between electrons and magnetic monopoles. Often when one theory is Higgsed and becomes weaker, its dual is confining and becomes stronger.

The dual theory also has magnetic baryon operators defined by

$$\begin{aligned} b^{i_1,\ldots,i_{N_f-N_c}} &= q^{n_1 i_1} \ldots q^{n_{N_f-N_c} i_{N_f-N_c}} \epsilon_{n_1,\ldots,n_{N_f-N_c}} \ , \\ \tilde{b}_{i_1,\ldots,i_{N_f-N_c}} &= \tilde{q}_{n_1 i_1} \ldots \tilde{q}_{n_{N_f-N_c} i_{N_f-N_c}} \epsilon^{n_1,\ldots,n_{N_f-N_c}} \ , \end{aligned} \tag{2.32}$$

and the singlet mesons $M^i_j$ are the magnetic analogs of the composite mesons $Q^i\tilde{Q}_j$ of the electric theory. Other operators can also be mapped from the electric theory to the magnetic theory.

The one-loop $\beta$ function of the dual theory is given by

$$\beta(\tilde{g}) \propto -\tilde{g}^3(3\tilde{N}_c - N_f) = -\tilde{g}^3(2N_f - 3N_c) . \tag{2.33}$$

We see that the magnetic theory is not asymptotically free in the range $N_f < 3N_c/2$. In this regime Seiberg's duality predicts that the strongly interacting electric theory is in fact asymptotically free. Since the weakly coupled variables in this case are the magnetic variables, we refer to the electric theory as being in a free magnetic phase.

In the range $N_f > 3N_c/2$ the magnetic theory is asymptotically free but just like in the electric case, when $N_f$ is sufficiently close to $3N_c/2$ it describes weakly interacting magnetic quarks and gluons (as well as the mesons $M$) in the IR. As $N_f$ is increased, the coupling in the IR increases and the electric and magnetic descriptions provide complimentary pictures of the non-abelian Coulomb phase. As $N_f$ increases the electric description becomes more weakly coupled (and thus more useful to study) while the magnetic theory becomes more strongly coupled and vice-versa.

Having two different descriptions can be very useful if one theory is strongly coupled and the other is weakly coupled, you can calculate non-perturbative effects in one theory by simply doing perturbative calculations in other theory. To summarise, Seiberg's duality allows the study of the low energy dynamics of the electric theory in the regime $N_c + 1 < N_f < \frac{3}{2}N_c$ by passing to the magnetic variables.

### 2.3.2 Consistency of Seiberg's Duality

The original SQCD examples constructed by Seiberg have been generalised in different directions, and there exists many additional examples of the basic phenomenon, see for example [42].

No proof of Seiberg's duality exists but there is a lot of evidence supporting it. The duality is not yet understood in the context of gauge theory and we do not know how to perform a transformation from the electric to the magnetic path integral. There are three non-trivial consistency tests of Seiberg's duality:

1. The global anomalies of the original quarks and gauginos match the 't Hooft anomaly matching conditions [43] (see Figure 2.1) of the dual quarks, dual gauginos and dual mesons.

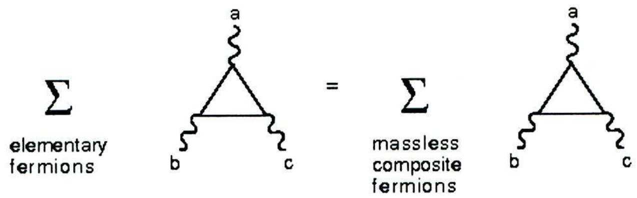


Figure 2.1: The t' Hooft anomaly matching condition

2. The two theories have the same moduli spaces of vacua and the gauge invariant operators match.

3. Integrating out a flavour in the original theory results in an $SU(N_c)$ theory with $N_f - 1$ flavours, which has a dual theory with $SU(N_f - N_c - 1)$ and $N_f - 1$ flavours in agreement with the expected result from a Seiberg duality.

In these tests the classical theories are different and only the quantum theories are equivalent.

# Chapter 3

# Metastable SUSY breaking Models

Intriligator, Seiberg and Shih (ISS) discovered [15] that simple field theories such as $\mathcal{N} = 1$ SQCD with light massive flavours possess metastable vacua with the lifetime of the order of the Universe. It opened up new avenues in model building, see for example [44]. In this chapter we briefly review some of these models: the ISS model [15] and some generalisations [16], [17] and [18]. Later on in Section 4.4 we will see how some of these models are embedded in string theory.

## 3.1 The ISS Model

We consider the model discussed in Section 2.2 namely SQCD with $SU(N_c)$ gauge group, $N_f$ light massive flavours $Q$, $\tilde{Q}$, dynamical scale $\Lambda$ and superpotential

$$W = \mathrm{Tr}\; mM \ , \tag{3.1}$$

where $M$ is defined in (2.23) and $m$ is an $N_f \times N_f$ non-degenerate mass matrix for the quarks. We are interested in the regime where the masses $m_i$ are small, much smaller than $\Lambda$, and roughly the same size:

$$m_i \ll |\Lambda| \ ; \frac{m_i}{m_j} \sim 1 \ . \tag{3.2}$$

The Witten index[1] predicts that the theory with superpotential (3.1) has $N_c$ supersymmetric vacua. These supersymmetric vacua are at

$$\langle M \rangle = \left(\Lambda^{3N_c - N_f} \det m\right)^{\frac{1}{N_c}} \frac{1}{m} \ . \tag{3.3}$$

The Seiberg magnetic dual theory is an $SU(N \equiv N_f - N_c)$ gauge theory with $N_f$ dual magnetic flavours $q_f$, $\tilde{q}_g$ and magnetic scale $\tilde{\Lambda}$, with superpotential

$$W_{mag} = h\mathrm{Tr}\ Mq\tilde{q} - h\mu^2 \mathrm{Tr}\ M \ . \tag{3.4}$$

We will work in the free magnetic range,

$$N_c + 1 \leq N_f < \frac{3}{2} N_c \ , \tag{3.5}$$

so that the Seiberg dual theory is IR free. In this range corrections to the Kähler potential are small since the metric on the moduli space is smooth around the origin and we can take the canonical Kähler potential

$$K = \mathrm{Tr}\ q^\dagger q + \mathrm{Tr}\ \tilde{q}^\dagger \tilde{q} + \mathrm{Tr}\ M^\dagger M \ . \tag{3.6}$$

### 3.1.1 Non-Supersymmetric Metastable Vacua

We will solve the $F$-term and $D$-term equations for the magnetic superpotential (3.4) and see that it admits a non-supersymmetric stable vacuum.

The dual theory with superpotential (3.4) breaks supersymmetry[2] at tree-level due to the $F$-term of $M$:

$$F_{M_{ij}} = h\tilde{q}^{jc} q^i_c - h\mu^2 \delta^{ij} \ , \tag{3.7}$$

[1] In supersymmetric theories each state of non-zero energy is always paired with another of opposite statistics, the states may only make the transition from zero to non-zero energy or *vice-versa* in pairs. Thus the number of bosonic zero-energy states minus the number of fermionic zero states does not change as the parameters of the theory vary; this difference is known as the Witten index [14]. Formally this index is Tr $(-1)^F$, where $F$ is the fermion number, the pairing of states insures that the trace can receive no contribution from states of non-zero energy. If the Witten index is non-zero then there must be some states of zero energy, and so supersymmetry cannot be broken.

[2] We can easily generalise this to the case of arbitrary quark masses $m_i \ll |\Lambda|$.

which cannot all vanish since $\delta^{ij}$ has rank $N_f$ while $\tilde{q}^{jc}q^i_c$ only has rank $N_f - N_c$. Supersymmetry is thus spontaneously broken by the so called rank condition. The potential is minimised along a classical moduli space of vacua which, up to global symmetries, is given by

$$M = \begin{pmatrix} 0 & 0 \\ 0 & M_0 \end{pmatrix}, \qquad q = \begin{pmatrix} q_0 \\ 0 \end{pmatrix}, \qquad \tilde{q}^T = \begin{pmatrix} \tilde{q}_0 \\ 0 \end{pmatrix}, \tag{3.8}$$

where $M_0$ is an $(N_f - N) \times (N_f - N)$ matrix, and $q_0$ and $\tilde{q}_0$ are $N \times N$ matrices satisfying $\tilde{q}_0 q_0 = \mu^2 \mathbb{I}_N$. Examining the one-loop effective potential around the vacua (3.9) using the Coleman-Weinberg effective potential [15] shows that all pseudomoduli (classical flat directions not corresponding to Goldstone bosons) are lifted, and the point in the moduli space of vacua with maximal unbroken global symmetry (up to unbroken flavour rotations) is

$$M_0 = 0, \qquad q_0 \tilde{q}_0 = \mu^2 \mathbb{I}_N \,. \tag{3.9}$$

This gives a minimum of the one-loop effective potential at

$$V_{meta} = (N_f - N)|h^2 \mu^4| \,. \tag{3.10}$$

The coupling is (marginally) irrelevant in the infrared and so the one-loop corrections dominate over higher order corrections.

### 3.1.2 Supersymmetric Vacua

We see here that supersymmetry is restored at high values in field space, this makes the non-supersymmetric vacua discovered in the previous section only metastable.

The $SU(N)$ gauge dynamics are IR free in the range $N_f > 3N$ and hence not relevant in the small field region however they become important in the large field region. In fact, it leads to the appearance of the $N_c$ supersymmetric vacua predicted by the Witten index. In the large field region of $M$, $|\langle hM \rangle| \gg |\mu|$, the $N_f$ flavours are very massive and we can integrate them out below the scale $|\langle hM \rangle|$ to obtain pure $SU(N)$ SYM dynamics with scale $\Lambda_{SYM}$. The complete low energy superpotential, including a non-perturbative correction coming from $SU(N)$ gaugino condensation is

$$W_{low} = N(h^{N_f}\Lambda_{Landau}^{-(N_f-3N)} \det M)^{\frac{1}{N}} - h\mu^2 \mathrm{Tr}\ M\ . \tag{3.11}$$

The equation of motion for $M$ gives the $N_c$ predicted supersymmetric vacua at

$$\langle hM \rangle = \Lambda_{Landau}\epsilon^{2N(N_f-N)}\mathbb{I}_{N_f}\ , \tag{3.12}$$

where

$$\epsilon \equiv \frac{\mu}{\Lambda_{Landau}}\ . \tag{3.13}$$

In the regime $\epsilon \ll 1$, the vevs are much smaller than the Landau pole scale $\Lambda_{Landau}$, and the analysis can be trusted.

To summarise the results, we found for $\Lambda_{Landau}$ approaching infinity with $\mu$ fixed, the theory breaks supersymmetry. For $\Lambda_{Landau}$ large but finite, a supersymmetric vacuum comes in from infinity. Since there are supersymmetric vacua elsewhere in the field space, the non-supersymmetric vacua are at a local minimum and are only metastable. We will now estimate the lifetime of these metastable vacua.

### 3.1.3 Lifetime of the Metastable Vacua

We found non-supersymmetric vacua at

$$M = 0, \qquad q = \tilde{q}^T = \mu\mathbb{I}_N, \qquad V_{meta} = (N_f - N)|h^2\mu^4|\ , \tag{3.14}$$

and supersymmetric vacua at

$$\langle hM \rangle = \Lambda_{Landau}\epsilon^{2N(N_f-N)}\mathbb{I}_{N_f}\ , \qquad V_{SUSY} = 0\ . \tag{3.15}$$

Since $N_f > 3N$ the supersymmetric minima sit at $|\langle hM \rangle| \gg |\mu|$, hence very far from the non-supersymmetric minimum. This separation and the height of the potential barrier ensures that the metastable state is parametrically long lived [15] possibly having a lifetime of the order of the Universe. The tunnelling probability is $\sim \exp(-S_{bounce})$, where $S_{bounce} \sim \Delta M^4/V_{meta}$, with $\Delta M$ the separation in field space between the metastable and the supersymmetric vacua. For small masses $S_{bounce}$ is large and the metastable DSB vacua can be made arbitrarily long lived.

In conclusion, we have found that $\mathcal{N} = 1$ SQCD with light massive flavours in the regime (3.5) admits metastable vacua with the lifetime of the Universe.

## 3.2 Metastable vacua in SQCD with Adjoint Matter

In this section we present a generalisation of the ISS model by Girardello *et al* [16]. We first give the description of the model and its magnetic dual then we present the non-supersymmetric vacua and the supersymmetric vacua, and we show that the lifetime of the non-supersymmetric vacua can be made arbitrarily large.

### 3.2.1 $\mathcal{N} = 1$ SQCD with Adjoint Matter

Consider $\mathcal{N} = 1$, $SU(N_c)$ SQCD with $N_f$ massive quarks, $Q$, $\tilde{Q}$ in the fundamental and antifundamental representations of the gauge group respectively, a massive adjoint field $X$, and superpotential

$$W_{el} = \frac{g_X}{3}\mathrm{Tr}\ X^3 + \frac{m_X}{2}\mathrm{Tr}\ X^2 + \lambda_X \mathrm{Tr}\ X \ , \qquad (3.16)$$

where $\lambda_X$ is a Lagrange multiplier enforcing the tracelessness of $X$. The coefficient of the beta function is $b = 2N_c - N_f$ so the theory is asymptotically free in the range $N_f < 2N_c$ and it admits stable vacua for $N_f > N_c/2$ [45].

The Seiberg dual theory [45–47] is an $SU(\tilde{N} \equiv 2N_f - N_c)$ gauge theory with $N_f$ magnetic quarks $q$, $\tilde{q}$, a massive magnetic adjoint field $Y$ and two gauge singlets $M_1$, $M_2$ constructed out of the electric quarks by $M_1 = Q\tilde{Q}$

and $M_2 = QX\tilde{Q}$, with superpotential

$$W_{mag} = \frac{g_Y}{3}\text{Tr}\, Y^3 + \frac{m_Y}{2}\text{Tr}\, Y^2 + \lambda_Y \text{Tr}\, Y - \frac{m_Y}{2\mu^2}\text{Tr}\, M_1 q\tilde{q} - \frac{g_Y}{\mu^2}\text{Tr}\, M_2 qY\tilde{q}\,. \quad (3.17)$$

The coefficient of beta function is given by $b = (3\tilde{N} - N_f) - \tilde{N}$ and we will consider the range

$$\frac{N_c}{2} < N_f < \frac{2N_c}{3}\,, \quad (3.18)$$

where the magnetic theory is infrared free and admits stable vacua.

### 3.2.2 Non-Supersymmetric Metastable Vacua

We deform the superpotential (3.17) and by solving the corresponding equations of motion we find a non-supersymmetric vacuum in the region of small fields where the $SU(\tilde{N})$ gauge dynamics is decoupled. The gauge dynamics become relevant in the large field region where they restore supersymmetry via non perturbative effects.

We deform the electric superpotential (3.16) by

$$W_{el} = \frac{g_X}{3}\text{Tr}\, X^3 + \frac{m_X}{2}\text{Tr}\, X^2 + \lambda_X \text{Tr}\, X + \lambda_Q \text{Tr}\, QX\tilde{Q} + m_Q \text{Tr}\, Q\tilde{Q} + h\text{Tr}\, (Q\tilde{Q})^2\,, \quad (3.19)$$

The terms $\lambda_Q \text{Tr}\, QX\tilde{Q}$ and $m_Q \text{Tr}\, Q\tilde{Q}$ are standard deformations of the electric superpotential [47]. The last term can be thought as originating from a massive adjoint field with a mass much higher than the scale of the theory that has been integrated out.

Consider that the duality relations are still valid after these deformations and that the deformations in the electric theory map to mesons in the magnetic theory [16] so that the superpotential of the magnetic theory is now

$$\begin{aligned} W_{magn} \;=\; & \frac{g_Y}{3}\text{Tr}\, Y^3 + \frac{m_Y}{2}\text{Tr}\, Y^2 + \lambda_Y \text{Tr}\, Y + h_1 \text{Tr}\, M_1 q\tilde{q} + h_2 \text{Tr}\, M_2 q\tilde{q} + \\ & h_3 \text{Tr}\, M_1 qY\tilde{q} - h_1 m_1^2\, \text{Tr}\, M_1 - h_2 m_2^2\, \text{Tr}\, M_2 + m_3\, \text{Tr}\, M_1^2\, . \end{aligned} \quad (3.20)$$

We then have a $\mathcal{N} = 1$ supersymmetric $SU(\tilde{N})$ gauge theory with $N_f$ magnetic flavours $(q, \tilde{q})$, an adjoint field $Y$, and two gauge singlet mesons $M_1$,$M_2$, with canonical Kähler potential. As in Section 3.1 we can take a canonical form for the Kähler potential.

The dual theory breaks supersymmetry at tree-level due to the $F$-term of $M_2$:

$$F_{M_2} = h_2 q\tilde{q} - h_2 m_2^2 \delta_{ij} = 0 \ , \tag{3.21}$$

which cannot all vanish since $\delta_{ij}$ has rank $N_f$, while $q\tilde{q}$ only has rank $\tilde{N}$. This is the rank condition of [15]. We solve the other $F$-term and $D$-term equations, choosing $Y$ to be diagonal finding

$$q = \begin{pmatrix} m_2 e^{\theta} \mathbb{I}_{\tilde{N}} \\ 0 \end{pmatrix}, \qquad \tilde{q}^T = \begin{pmatrix} m_2 e^{-\theta} \mathbb{I}_{\tilde{N}} \\ 0 \end{pmatrix}, \tag{3.22}$$

$$\lambda_Y = \frac{h_3 h_1 m_2^2}{2m_3}\left(m_2^2 - m_1^2\right) - \frac{m_Y^2}{g_Y}\left(1 - \frac{h_3^2 m_2^4}{2m_3 m_Y}\right)^2 \frac{n_1 n_2}{(n_1 - n_2)^2}\ , \tag{3.23}$$

$$\langle Y \rangle = \begin{pmatrix} y_1 \mathbb{I}_{n_1} & 0 \\ 0 & y_2 \mathbb{I}_{n_2} \end{pmatrix}, \tag{3.24}$$

where $y_1 = -\frac{m_Y - \frac{h_3^2 m_2^4}{2m_3}}{g_Y}\frac{n_2}{n_1 - n_2}$, $y_2 = \frac{m_Y - \frac{h_3^2 m_2^4}{2m_3}}{g_Y}\frac{n_1}{n_1 - n_2}$.

We choose the vacuum in which $\langle Y \rangle = 0$ so that the magnetic gauge group is not broken by the adjoint field[3]. The vevs for the mesons are then given by

$$\langle M_1 \rangle = \begin{pmatrix} \frac{h_1}{2m_3}(m_1^2 - m_2^2)\ \mathbb{I}_{\tilde{N}} & 0 \\ 0 & \frac{h_1 m_1^2}{2m_3}\ \mathbb{I}_{N_f - \tilde{N}} \end{pmatrix} \tag{3.25}$$

$$\langle M_2 \rangle = \begin{pmatrix} -\frac{h_1^2}{2h_2 m_3}(m_1^2 - m_2^2)\ \mathbf{1}_{\tilde{N}} & 0 \\ 0 & \mathcal{X} \end{pmatrix} \tag{3.26}$$

where $\mathcal{X}$ is an $N_f - \tilde{N} \times N_f - \tilde{N}$ matrix undetermined at the classical level.

We examine the one-loop effective potential around the vacua $\langle Y \rangle = 0$, (3.22), (3.25), (3.26) using the Coleman-Weinberg effective potential. This shows [16], that all the pseudomoduli get positive masses, and there is a non-supersymmetric stable vacuum at $\theta = 0$ and $\mathcal{X} \neq 0$.

[3]Other choices for $\langle Y \rangle$ with $n_1 \neq 0 \neq n_2$ do not change the tree-level potential energy of the vacua.

### 3.2.3 Supersymmetric Vacuum

Supersymmetry is restored in the region of large field space via non perturbative effects. The non-supersymmetric vacuum discovered in the the previous section is a metastable state of the theory which decays into a supersymmetric vacuum.

We first integrate out the two massive fields $M_1$, $Y$ in the superpotential (3.20) using their equations of motion tuning $\lambda_Y$ in a way such that the gauge group $SU(\tilde{N})$ is not broken by the adjoint. After integrating out the adjoint field $Y$ the scale matching condition reads

$$\tilde{\Lambda}^{2\tilde{N}-N_f} = \tilde{\Lambda}_{adj}^{3\tilde{N}-N_f} m_Y^{-\tilde{N}} , \tag{3.27}$$

where $m_Y$ is the mass of the adjoint field. The equation of motion for the meson $M_1$ then gives

$$M_1 = \frac{h_1}{2m_3} \left(2m_1^2 - q\tilde{q}\right) . \tag{3.28}$$

The corresponding superpotential is given by

$$W_{int} = \mathrm{Tr} \left( \frac{h_1^2}{4m_3} \left(2m_1^2 q\tilde{q} - (q\tilde{q})^2\right) + h_2 M_2 q\tilde{q} - h_2 m_2^2 M_2 \right) . \tag{3.29}$$

Consider that the vev $\langle h_2 M_2 \rangle$ is large[4], we then integrate out the massive flavours $q$, $\tilde{q}$. The scale matching condition for integrating out the flavours is

$$\Lambda_{SYM}^{3\tilde{N}} = \tilde{\Lambda}_{adj}^{3\tilde{N}-N_f} \det\left(h_2 M_2\right) = \tilde{\Lambda}^{2\tilde{N}-N_f} \det\left(h_2 M_2\right) m_Y^{\tilde{N}} . \tag{3.30}$$

The low energy effective $SU(\tilde{N})$ superpotential gets a non-perturbative contribution from the gauge dynamics related to the gaugino condensation proportional to the low energy scale $\Lambda_{SYM}$

$$W = \tilde{N} \Lambda_{SYM}^3 . \tag{3.31}$$

We write this non-perturbative correction to the low energy effective superpotential with scale $\Lambda_{SYM}$ in terms of the macroscopic scale $\tilde{\Lambda}$ using the scale

[4] We neglect the contribution from the couplings.

matching condition (3.30). The low energy effective superpotential is then

$$W_{Low} = \tilde{N}\left(\tilde{\Lambda}^{2\tilde{N}-N_f}\det(h_2 M_2)\right)^{\frac{1}{\tilde{N}}} m_Y - m_2^2 h_2 \mathrm{tr}\ M_2 \tag{3.32}$$

We can use the equation of motion of $M_2$ to find the supersymmetric minima which are at

$$h_2\langle M_2\rangle = \tilde{\Lambda}\epsilon^{\frac{\tilde{N}}{N_f-\tilde{N}}}\xi^{\frac{\tilde{N}}{N_f-\tilde{N}}}\mathbb{I}_{N_f} = m_2\left(\frac{1}{\epsilon}\right)^{\frac{N_f-2\tilde{N}}{N_f-\tilde{N}}}\xi^{\frac{\tilde{N}}{N_f-\tilde{N}}}\mathbb{I}_{N_f}\ , \tag{3.33}$$

where $\epsilon = \frac{m_2}{\tilde{\Lambda}}$ is a dimensionless parameter which can be made parametrically small and $\xi = \frac{m_2}{m_Y}$ is a dimensionless finite parameter which does not spoil our estimation of the supersymmetric vacuum in the range $\epsilon < \frac{1}{\xi}$.

The hierarchy of scales is

$$m_2 \ll h_2\langle M_2\rangle \ll \tilde{\Lambda}\ , \tag{3.34}$$

which justifies neglecting the contribution from the couplings to the mass terms of the quarks in (3.29). It also shows that the evaluation of the supersymmetric vacuum is reliable because the scale of $h_2\langle M_2\rangle$ is well below the Landau pole $\tilde{\Lambda}$.

### 3.2.4 Lifetime of the Metastable Vacua

We would like to estimate the lifetime of the metastable vacuum. At semiclassical level the decay probability is proportional to $\exp(-S_{bounce})$ where $S_{bounce}$ is the bounce action from the non-supersymmetric vacuum to a supersymmetric one. The supersymmetric minimum and non-supersymmetric minimum are not of the same order and so the thin wall approximation of [15] cannot be used. The decay rate is approximated to be [16]

$$S \sim \left(\left(\frac{1}{\epsilon}\right)^{\frac{N_f-2\tilde{N}}{N_f-\tilde{N}}}\xi^{\frac{\tilde{N}}{N_f-\tilde{N}}}\right)^4 \sim \left(\frac{1}{\epsilon}\right)^{4\frac{N_f-2\tilde{N}}{N_f-\tilde{N}}}\ . \tag{3.35}$$

This rate can be made parametrically large in the regime $\epsilon \ll 1$ giving the metastable vacua parametrically long life.

To summarise we have found that in $SU(N_c)$ SQCD with two adjoint chiral fields and mesonic deformations there is also a metastable non-supersymmetric vacuum with the lifetime of the order of the Universe.

## 3.3 Metastable Vacua in Deformed SQCD

In this section we consider the model of metastable symmetry breaking of Giveon and Kutasov [17]. We will see that it admits a metastable vacuum which is very similar to that of the ISS model.

### 3.3.1 Deformed SQCD

Consider $\mathcal{N}=1$ SQCD with gauge group $SU(N_c)$ with $N_f > N_c$ light massive flavours $Q,\ \tilde{Q}$ in the fundamental and antifundamental representations of the gauge group respectively, and superpotential

$$W_{el} = \frac{\alpha}{2}\mathrm{Tr}\ (\tilde{Q}Q)^2 - m\mathrm{Tr}\ \tilde{Q}Q = \frac{\alpha}{2}\mathrm{Tr}\ M^2 - m\mathrm{Tr}\ M\ . \tag{3.36}$$

The magnetic dual description is an $SU(N_f - N_c)$ gauge theory with $N_f$ dual magnetic flavours $q,\ \tilde{q}$, gauge singlet $M \equiv Q\tilde{Q}$ and magnetic superpotential

$$W_{mag} = \frac{1}{\Lambda}\mathrm{Tr}\ \tilde{q}Mq + \frac{\alpha}{2}\mathrm{Tr}\ M^2 - m\mathrm{Tr}\ M\ . \tag{3.37}$$

The magnetic superpotential is quadratic in $M$ and we can integrate this field out via its equation of motion. This gives the superpotential

$$W_{mag} = -\frac{1}{\alpha\Lambda}\left[\frac{1}{2\Lambda}\mathrm{Tr}(\tilde{q}q)^2 - m\mathrm{Tr}(\tilde{q}q)\right]\ . \tag{3.38}$$

Note that this superpotential has the same qualitative features as the electric superpotential (3.36) where neither superpotential contains a gauge singlet.

Conversely, we may integrate in (see [48]) a gauge singlet field defined by $N = \tilde{q}q$ in analogy with the magnetic meson $M$, to the electric superpotential

(3.36) obtaining the superpotential

$$W_{el} = -\frac{1}{\Lambda}\text{Tr}\ \tilde{Q}^i N_i^j Q_j - \frac{\alpha_e}{2}\text{Tr}\ N^2 + m_e \text{Tr}\ N\ . \qquad (3.39)$$

Note that this superpotential has the same qualitative features as the magnetic superpotential (3.37) with both containing gauge singlet fields.

### 3.3.2 Supersymmetric Vacua

First we will find the $N_f - N_c$ supersymmetric vacua of the magnetic theory with superpotential (3.37) predicted by the Witten index. The $F$-term conditions are

$$M_j^i q^j = 0, \qquad \tilde{q}_i M_j^i = 0, \qquad \frac{1}{\Lambda}\tilde{q}_i q^j = m\delta_i^j - \alpha M_i^j\ . \qquad (3.40)$$

These imply $M$ satisfies

$$mM = \alpha M^2\ . \qquad (3.41)$$

$M$ can be chosen to be diagonal, then (3.41) implies that its eigenvalues can take only two values, 0 and $\frac{m}{\alpha}$. Thus, $M$ takes the form

$$M = \begin{pmatrix} 0 & 0 \\ 0 & \frac{m}{\alpha}\mathbb{I}_{N_f-k} \end{pmatrix}, \qquad (3.42)$$

where $k = 0, 1, 2, \ldots, N_f$. The $F$-term equations then give

$$\tilde{q}q = \begin{pmatrix} m\Lambda\mathbb{I}_k & 0 \\ 0 & 0 \end{pmatrix}. \qquad (3.43)$$

Since the rank of $\tilde{q}q$ is at most $N_f - N_c$, we must have $k \leq N_f - N_c$.

### 3.3.3 Metastable Vacua

In this section by decomposing the supersymmetric vacua we will see that this theory admits non-supersymmetric metastable vacua. As in [15], we will do our analysis in the range $N_f < \frac{3}{2}N_c$ where the magnetic theory is infrared free and can be thought of as the effective low energy description of the asymptotically free electric gauge theory. We make a change of variables $M = \sqrt{a}\Lambda_e\Phi$, in these variables the magnetic superpotential (3.37) takes the form

$$W_{\text{mag}} = h\text{Tr}\ \tilde{q}\Phi q - h\mu^2\text{Tr}\ \Phi - \frac{1}{2}h^2\mu_\phi\text{Tr}\ \Phi^2 = \frac{1}{\Lambda}\tilde{\text{Tr}}\ qMq + \frac{1}{2}\alpha\text{Tr}\ M^2 - \text{Tr}\ mM\ , \tag{3.44}$$

and the supersymmetric vacua take the form

$$h\Phi = \begin{pmatrix} 0 & 0 \\ 0 & \frac{\mu^2}{\mu_\phi}\mathbb{I}_{N_f-k} \end{pmatrix}, \tag{3.45}$$

$$\tilde{q}q = \begin{pmatrix} \mu^2\mathbb{I}_k & 0 \\ 0 & 0 \end{pmatrix}. \tag{3.46}$$

As in the other two models discussed in Section 3.1 and Section 3.2, we may take the Kähler potential to be canonical near the origin of field space

$$K = \text{Tr}q^\dagger q + \text{Tr}\tilde{q}^\dagger\tilde{q} + \text{Tr}\Phi^\dagger\Phi + \cdots\ . \tag{3.47}$$

To make the metastable vacua apparent we decompose the supersymmetric vacua as follows

$$h\Phi = \begin{pmatrix} 0 & 0 & 0 \\ 0 & h\Phi_n & 0 \\ 0 & 0 & \frac{\mu^2}{\mu_\phi}\mathbb{I}_{N_f-k-n} \end{pmatrix}, \tag{3.48}$$

$$\tilde{q}q = \begin{pmatrix} \mu^2\mathbb{I}_k & 0 & 0 \\ 0 & \tilde{\varphi}\varphi & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{3.49}$$

Here $\varphi^t$ and $\tilde{\varphi}$ are $n \times (N_f - N_c - k)$ dimensional matrices and $\Phi_n$ is an $n \times n$ matrix. $\varphi$ and $\tilde{\varphi}$ correspond to $n$ flavours in the fundamental and antifundamental representations of the broken gauge group $SU(N_f - N_c - k)$ respectively.

We perform the analysis in the range

$$\Lambda_m \gg \mu \gg \mu_\phi \tag{3.50}$$

where the first inequality is as in [15], and the second implies that the term proportional to $\mu_\phi$ is a small perturbation of the superpotential considered in [15]. Since in general the expectation value of $\Phi$ (3.48) can be large in the regime (3.50), we will discuss separately the cases $n = N_f - k$ and $n < N_f - k$ starting with the former.

### Metastable vacua with $n = N_f - k$

We see here that when we gauge part of the gauge group by giving a vev of $k$ of the magnetic quarks, the theory reduces to a very similar computation to that of [15].

We give a vev $\mu$ to $k$ of the magnetic quarks in (3.49). The low energy effective theory is an $SU(N_f - N_c - k)$ gauge theory with $N_f - k = n$ light flavours and scale $\Lambda_{low}$, related to the macroscopic scale by

$$\Lambda_m^{3(N_f-N_c)-N_f} = \mu^{2k}\Lambda_{low}^{3(N_f-N_c-k)-n} . \tag{3.51}$$

The $SU(N_f - N_c - k)$ gauge theory is infrared free in the regime $N_f < \frac{3}{2}N_c$ and in the range (3.50) the hierarchy of scales is

$$\mu \ll \Lambda_{low} \ll \Lambda_m . \tag{3.52}$$

Since we are at a scale $\mu$, well below the macroscopic scale $\Lambda_m$, we can use the low energy theory described by the $SU(N_f - N_c - k)$ gauge group with $n$ light flavours.

We now find the minimum of the scalar potential for $\Phi_n$, $\varphi$, $\tilde{\varphi}$ near the origin of field space. There are two contributions to the potential, the first is the tree level potential that follows from (3.44) and (3.47); while the second is the one-loop potential Coleman-Weinberg potential, which is the same as the one computed in [15]. The one-loop potential has the form

$$\frac{V}{|h|^2} = |\Phi_n\varphi|^2 + |\tilde{\varphi}\Phi_n|^2 + |\tilde{\varphi}\varphi - \mu^2 \mathbb{I}_n + h\mu_\phi \Phi_n|^2 + b|h\mu|^2 \mathrm{Tr}\ \Phi_n^\dagger \Phi_n \ , \quad (3.53)$$

where $b$ is a non-negative constant. We extremise the potential to locate the minimum at $\varphi = \tilde{\varphi} = 0$ finding

$$h\Phi_n = \frac{\mu^2 \mu_\phi}{|\mu_\phi|^2 + b|\mu|^2} \mathbb{I}_n \simeq \frac{\mu^2 \mu_\phi}{b|\mu|^2} \mathbb{I}_n \ , \quad (3.54)$$

which gives a minimum of the scalar potential at

$$V \simeq n|h\mu^2|^2 \ . \quad (3.55)$$

Analogously to the previous examples, the lifetime of the metastable vacua can be made parametrically large.

### Metastable vacua with $n < N_f - k$

In this case, as in the previous case, we see how the gauge dynamics brings about a metastable vacuum. We give masses to $N_f - k - n$ of the magnetic flavours and vevs to $k$ flavours. The masses are much larger than the vevs in the range (3.50) so first we give mass and then expectation values. In order that the Kähler potential is under control we take

$$\frac{\mu^2}{\mu_\phi} \ll h\Lambda_m \ . \quad (3.56)$$

We start with the magnetic theory with gauge group $SU(N_f - N_c)$, and give masses $\frac{\mu^2}{\mu_\phi}$ to $N_f - k - n$ of the flavours in (3.49). We then give expectation value $\mu$ to $k$ of the flavours in (3.49). The low energy effective theory remains weakly coupled throughout this process [17], hence we can neglect the gauge dynamics and come to the same conclusion as in the case $n = N_f - k$.

In conclusion, deformed SQCD in the regime (3.50) has metastable vacua with a lifetime of the order of the Universe.

## 3.4 Other Metastable Vacua

In this section we attempt to sketch some of the ideas of [18], postponing a thorough discussion until Section 5.4 where will will have all the necessary tools. There seems to be no simple field theory describing this model that has a finite degree of freedom and we have to turn to string theory to describe it. The idea is that we break supersymmetry by adding branes into a string theory background to break half the supersymmetry and antibranes breaking a different half. The attraction between the branes and antibranes gives the lifetime of the metastable non-supersymmetric vacua.

A Calabi-Yau background preserves $\mathcal{N} = 2$ supersymmetry. In the low energy limit wrapping $N$ D5 branes on a $\mathbb{P}^1$ cycle of a resolved conifold singularity of the Calabi-Yau manifold further breaks supersymmetry to $\mathcal{N} = 1$. The theory described by the $N$ wrapped branes is a pure Yang-Mills theory with gauge group $SU(N)$ and superpotential

$$W(S) = \alpha S + \frac{1}{2\pi i} N S(\log(S/\Lambda_0^3) - 1) \ , \tag{3.57}$$

where $S$ is the glueball superfield given by $S = \frac{1}{32\pi^2}\mathrm{Tr}\mathcal{W}_\alpha \mathcal{W}^\alpha$, and $\alpha$ is the bare gauge coupling constant defined at the cutoff scale $\Lambda_0$ by

$$\alpha(\Lambda_0) = -\frac{\theta}{2\pi} - i\frac{4\pi}{g_{\mathrm{YM}}{}^2(\Lambda_0)} \ . \tag{3.58}$$

Extremising $W_{eff}$, we find $N$ supersymmetric vacua at

$$\langle S\rangle = \Lambda_0^3 \exp(-\frac{2\pi i\alpha}{N}) \exp(\frac{2\pi i k}{N}) \qquad k = 1, \ldots, N \ . \tag{3.59}$$

We will see in Section 5.4 that in the large $N$ limit, the $N$ D-branes wrapped on the $S^2$ cycles get replaced by $N$ units of flux through the $S^3$ of the deformed conifold. In the antibrane case the $N$ units of flux are negative and the superpotential is still given by (3.57) but with $N$ negative.

The case of $N$ wrapped antibranes should lead to the same minimum as (3.59) with $N$ negative, however $S$ is related to the size of the $S^3$ and this would give an unphysical infinitely sized $S^3$ in the weak coupling limit. The superpotential (3.57) is chosen to give the $\mathcal{N} = 1$ supersymmetry described by the D-branes and not the anti D-branes. If you like the choice of branes or antibranes depends on the choice of gauge in the Lagrangian and gives different $\mathcal{N} = 1$ supersymmetry. Having both branes and antibranes will totally break supersymmetry. The full discussion is postponed until Section 5.4 where a string theory treatment is given as there is no known simple field theory describing this example.

# Chapter 4

# Brane Configurations

In this chapter we will build up the necessary tools in order to study the brane configurations of the field theory models described in Chapters 2 and 3 and to understand how Seiberg duality works in the context of brane configurations. We work in IIA string theory and look at the low energy field theory described by systems of D-branes and NS5 branes starting with just a single D-brane. We discuss what limits need to be taken in order for our analysis to be valid. A basic knowledge of string theory and of D-branes is assumed, for reviews see [3, 49–54].

## 4.1 Branes

### 4.1.1 The Low Energy Effective Field Theory of D-branes

A D-brane is defined by the property that fundamental strings can end on it. A Dp-brane stretched in the $x^{1,\dots,p}$ directions carries Ramond-Ramond (RR) charge andpreserves supercharges of the form $\epsilon_L Q_L + \epsilon_R Q_R$, where

$$\epsilon_L = \Gamma^0 \Gamma^1 \cdots \Gamma^p \epsilon_R \ . \tag{4.1}$$

An anti Dp-brane carries the opposite RR charge and preserves the other half of the supercharges. An NS fivebrane stretched in the $x^{1,\dots,5}$ directions is charged under the magnetic $B$-field and in IIA preserves

$$\begin{aligned} \epsilon_L = & \ \Gamma^0\Gamma^1\Gamma^2\Gamma^3\Gamma^4\Gamma^5\epsilon_L \\ \epsilon_R = & \ \Gamma^0\Gamma^1\Gamma^2\Gamma^3\Gamma^4\Gamma^5\epsilon_R \ , \end{aligned} \tag{4.2}$$

while in IIB it preserves

$$\begin{aligned} \epsilon_L = & \ \Gamma^0\Gamma^1\Gamma^2\Gamma^3\Gamma^4\Gamma^5\epsilon_L \\ \epsilon_R = & \ -\Gamma^0\Gamma^1\Gamma^2\Gamma^3\Gamma^4\Gamma^5\epsilon_R \ , \end{aligned} \tag{4.3}$$

The low energy effective field theory on a single Dp-brane stretched in $p+1$ dimensions is a $p+1$ dimensional field theory with sixteen supercharges. If we examine the massless spectrum of a single Dp-brane, we see that it contains a $p+1$ dimensional $U(1)$ gauge field $A_a(x^b)$, $9-p$ massless scalars $X^I(x^a)$ ($I = p+1, \cdots, 9$, $a = 0, \cdots, p$) for each normal direction to the brane, and fermions required by supersymmetry. The bosonic part of the low energy worldvolume action is

$$S = \frac{1}{g_{SYM}^2} \int d^{p+1}x \left( \frac{1}{4} F_{ab}F^{ab} + \frac{1}{l_s^4} \partial_a X^I \partial^a X_I \right) \ , \tag{4.4}$$

where the $U(1)$ gauge coupling $g_{SYM}$ is given by

$$g_{SYM}^2 = g_s l_s^{p-3} \ , \tag{4.5}$$

and $g_s$ and $l_s$ are the string coupling and string length respectively.

We wish to study the low energy effective field theory on the brane, in order to do this we must decouple the effects of gravity and massive string modes. We can achieve this decoupling by taking the limit $l_s \longrightarrow 0$, holding $g_{SYM}$ fixed. In this limit we find the field theory is Super Yang-Mills in $p+1$ dimensions, in particular for $p = 3$, $g_{SYM}$ is given simply by $g_s$ and taking the limit $l_s \rightarrow 0$ we find $\mathcal{N} = 4$ SYM in $3+1$ dimensions.

When we have a stack of coincident Dp-branes the situation becomes more interesting. We again take the above limit in order to decouple the effects of gravity and massive string modes.

The low energy effective theory of a stack of $N_c$ coincident Dp-branes is a $U(N_c)$ SYM theory with sixteen supercharges. The scalars $X^I$ now become $N_c \times N_c$ matrices transforming in the adjoint representation of the $U(N_c)$ gauge group. The $N_c$ photons in the Cartan subalgebra of $U(N_c)$ and the diagonal components of the matrices $X^I$ correspond to open strings both of whose endpoints lie on the same brane. The charged gauge bosons and off-diagonal components of $X^I$ correspond to strings whose endpoints lie on different branes.

The bosonic part of the $9+1$ dimensional low energy Lagrangian is given by

$$\mathcal{L} = \frac{1}{4g_{SYM}^2} \mathrm{Tr}\ F_{mn}F^{mn}; \quad m, n = 0, 1, \cdots, 9\ , \tag{4.6}$$

where

$$F_{mn} = \partial_m A_n - \partial_n A_m - i[A_m, A_n]\ . \tag{4.7}$$

Upon dimensional reduction to $p+1$ dimensions this Lagrangian gives rise to kinetic terms for the gauge field $A_\mu$ and adjoint scalars $X^I$:

$$\mathcal{L}_{kin} = \frac{1}{g_{SYM}^2} \mathrm{Tr} \left( \frac{1}{4} F_{ab}F^{ab} + \frac{1}{l_s^4} \mathcal{D}_a X^I \mathcal{D}^a X_I \right)\ , \tag{4.8}$$

and potential for the adjoint scalars $X^I$,

$$V \sim \frac{1}{l_s^8 g_{SYM}^2} \sum_{I,J} \mathrm{Tr}\ [X^I, X^J]^2\ , \tag{4.9}$$

where the covariant derivative and field strength are defined in the usual way:

$$\begin{aligned} \mathcal{D}_a X^I &= \partial_a X^I - i[A_a, X^I]\ , \\ F_{ab} &= \partial_{[a} A_{b]} - i[A_a, A_b]\ . \end{aligned} \tag{4.10}$$

The Coulomb branch of the theory is described by the flat directions of the potential (4.9). The moduli space of vacua is parametrised by the eigenvalues of $X^I$:

$$x_i = \langle X_{ii} \rangle; \quad i = 1 \cdots, N_c\ , \tag{4.11}$$

which give the position of each of the $N_c$ branes. When the stack of $N_c$ D-branes are separated in space, the fundamental strings between each of the branes become stretched and the off-diagonal components of $X^I$ and the charged gauge bosons become massive. The gauge symmetry is broken from $U(N_c)$ to a product of $U(1)$s corresponding to $N_c$ copies of $U(1)$ field theories with each described by a single brane.

The masses of the gauge charged bosons and off-diagonal components of $X^I$ are determined by the separation of the branes

$$m_{ij} = \frac{1}{l_s^2}|x_i - x_j| \ . \tag{4.12}$$

If instead we have $n$ of the branes coinciding and the other $N_c - n$ still remaining separated, then some of the charged particles become massless and the gauge group is enhanced from $U(1)^{N_c}$ to $U(n) \times U(1)^{N_c - n}$.

### 4.1.2 Systems of D-branes

We need to construct more elaborate configurations of branes in order to describe matter multiplets. Branes can end on other branes and this opens up vast possibilities. One constraint on systems of branes is that they must obey the s-rule [55], a phenomenological rule stating that a brane configuration is not supersymmetric if an NS fivebrane and a D6-brane are connected by more than one $D4$-brane.

Consider the low energy effective field theory of a brane ending on another brane. It is a theory with eight supercharges similar to that of the infinitely extended brane. The light fields can be described in terms of four dimensional $\mathcal{N} = 2$ supersymmetric hypermultiplets and vectormultiplets with spin $\leq 1$: Consider a Dp-brane stretched in the $x^{0,1,\dots,p}$ directions ending, in the $x^p$ direction, on a $D(p+2)$-brane stretched in the $x^{0,1,\dots,p-1,p+1,p+2,p+3}$ directions and located at $x^p = 0$. The low energy $p+1$ dimensional gauge theory associated with the Dp-brane takes place on the infinitely stretched part of the brane as well as the half line $x^p \geq 0$.

A massless hypermultiplet is filled out by the three scalars corresponding to fluctuations of the Dp-brane in the normal direction which are transverse to the $D(p+2)$-brane $X^{p+1}, X^{p+2}, X^{p+3}$ and the $p$'th component of the Dp-brane gauge field $A_p$. A vectormultiplet is formed from the $6-p$ scalars describing fluctuations of the Dp-brane normal to the $D(p+2)$-brane $X^{p+4}, \dots, X^9$ and the gauge field $A_c$, $c = 0, 1, \dots, p-1$.

Now consider a a Dp-brane stretched in the $x^{0,1,\dots,p-1,6}$ directions and ending, in the $x^6$ direction, on an $NS5$-brane stretched in the $x^{0,1,\dots,5}$ directions. A hypermultiplet is filled out by the scalars $X^7, X^8, X^9$ and the sixth component of the Dp-brane gauge field $A_6$, and satisfies Dirichlet boundary conditions at $x^6 = 0$. A vectormultiplet is filled out by $6-p$ scalars $X^p, \dots, X^5$ and the components of the gauge field in the $x^{1,\dots,p-1}$ directions.

## 4.2 Brane Configurations and $\mathcal{N} = 2$ Theories

Brane configurations of $\mathcal{N} = 2$ supersymmetric gauge theories were first constructed in [55] in IIB string theory and are reviewed thoroughly in IIA string theory in [3]. We now show how to construct $\mathcal{N} = 2$ supersymmetric gauge theories in brane configurations in IIA string theory.

Consider two infinite $NS5$-branes stretched in the $x^{0,\dots,5}$ directions, separated by a distance $s_6$ in the $x^6$ direction, and located at the same point in $(x^7, x^8, x^9)$. Stretch, in the $x^6$ direction, $N_c$ D4-branes between the $NS5$ branes, where the D4-branes are stretched in the $x^{0,\cdots,3,6}$ directions.

The D4-branes are finite in extent in the $x^6$ direction. At large distance scales, much larger than $s_6$, the physics on the D4-branes looks $3+1$ dimensional. Excitations of the fourbranes can be thought of as fields living in the $4+1$ dimensional space $\mathbb{R}^{1,3} \times$ (interval) where the interval is the finite length between the $NS5$-branes. Depending on the boundary conditions at the ends of the fourbranes, the different fields do or do not give rise to light fields in $1+3$ dimensions. We now analyse the different possibilities.

The light excitations on a stack of $N_c$ $D4$-branes stretched to infinity in the $x^{0,1,2,3,6}$ directions are a $U(N_c)$ gauge field $A_d$, where $d = 0, 1, 2, 3, 6$, and five scalars in the adjoint representation of the $U(N_c)$ gauge group corresponding fluctuations in the directions transverse to the brane. When instead we have the situation above with $N_c$ fourbranes stretched on the interval between two $NS5$-branes, $X^7, X^8, X^9$ as well as $A_6$ satisfy Dirichlet boundary conditions on both ends of the interval. These fields give masses of $\mathcal{O}(1/s_6)$ to states in $3+1$ dimensions. However, we are interested in the distances far from $s_6$ where the effects are negligible. There is a vectormultiplet filled out by the $U(N_c)$ gauge field $A_\mu$, $\mu = 0, 1, 2, 3$ and the adjoint scalars $X^4, X^5$ which satisfy free boundary conditions on the interval.

The effective field theory of the brane configuration with $N_c$ D4-branes on the interval between two $NS5$-branes at a large distance from them is an $\mathcal{N} = 2$ supersymmetric gauge theory with gauge group $U(N_c)$ and no matter. The gauge coupling of the $4+1$ dimensional field theory on the fourbranes can be found from (4.5) and is given by $g_{D4}^2 = g_s l_s$. We can find the $3+1$ dimensional gauge coupling by Kaluza-Klein reduction on the interval:

$$\frac{1}{g^2} = \frac{L_6}{g_s l_s} \tag{4.13}$$

We need to consider distances much larger than $s_6$ so that the $\mathcal{N} = 2$ $U(N_c)$ gauge theory is $3+1$ dimensional. We also need to suppress the couplings of the light fields on the fourbranes to light fields living on the $NS5$-branes and to fields living in the bulk of spacetime. Thus, we consider the limit $g_s \longrightarrow 0$ and distances much larger than $l_s$.

The limit of small coupling also allows us to neglect the quantum corrections caused by the ends of the D4-branes bending the $NS5$-branes. The quantum rule governing brane configurations postulated in [3] is that when we have a system of a D4-brane stretched between an $NS5$ and an $NS5'$-brane and another D4-brane which ends on either of the NS-branes, then there is a repulsive force if the $D4$-branes are on the same side of the NS-brane, and an attractive force if the D4-branes are on different sides of the NS-brane.

### 4.2.1 Adding Fundamental Matter

To describe $U(N_c)$ $\mathcal{N} = 2$ SYM with matter in the fundamental representation we must add to the configuration described above either semi-infinite D4-branes or D6-branes. Consider adding $N_f$ semi-infinite fourbranes ending on the left $NS5$-brane, in the $x^6$ direction, from the left, and extending to infinity. This gives rise to $N_f$ hypermultiplets in the fundamental representation of $U(N_c)$ corresponding to strings stretched between the $N_c$ fourbranes and the $N_f$ semi-infinite fourbranes. By varying the locations of the semi-infinite D4-branes in the $(x^4, x^5)$ plane the strings between the $N_f$ semi-infinite D4-branes and $N_c$ D4-branes become stretched. This corresponds to masses for the fundamentals. In this way we can describe the Coulomb phase of the gauge theory.

To describe the Higgs phase we must replace the semi-infinite D4-branes with $N_f$ finite D4-branes with each ending on a D6-brane stretched in the $x^{0,\ldots,3,7,8,9}$ directions. The gauge theory interpretation of the Higgs branch is along the moduli space of vacua where some, or all, of the fundamentals $Q$, $\tilde{Q}$ get expectation values and the rank of the unbroken gauge group decreases. For $N_f \geq \sqrt{2}N_c$ the gauge group $U(N_c)$ can be completely Higgsed and the complex dimension of the corresponding branch of moduli space is $2N_cN_f - 2N_c^2$.

We can give mass to the flavours by separating the $N_f$ flavour D4-branes in the $(x^4, x^5)$ plane relative to the $N_c$ colour D4 branes. To enter the Higgs phase we start with two D6-branes in the same position in the $(x^4, x^5)$ plane, *i.e.* the flavours have the same mass. Each D6-brane is connected to the same $NS5$-brane by a D4-brane but we can still separate the D6-branes in the $x^6$ direction. We can connect the D4 branes between the $NS5$-brane and D6-branes leaving the D4-brane in two pieces. One stretched between the $NS5$-brane and the D6-brane and the other stretched between two D6-branes. A massless hypermultiplet comes from the D4-brane stretched between the two D6-branes, the scalars corresponding to displacements of the D4-brane along directions transverse to the D6-brane and the compact part of the gauge field. The relative separation of the two parts of the D4-brane gives the value of the vev for the fundamental flavour.

## 4.3 Brane Configurations and $\mathcal{N} = 1$ Theories

We would like to describe $\mathcal{N} = 1$ SQCD with matter in the fundamental representation of the gauge group and Seiberg's duality in the brane picture. Both were shown in [3, 10, 11]. To break supersymmetry to $\mathcal{N} = 1$ we start with the $\mathcal{N} = 2$ theory above with gauge group $SU(N_c)$ and $N_f$ fundamental hypermultiplets. We rotate one or more of the branes with respect to the others in order to further break the number of supercharges down to four. The rotation of the fivebranes is related to giving a mass to the adjoint field.

First we will see what happens if we rotate one or both of the $NS5$-branes or one or more of the D6-branes. We rotate the branes in the $(v, w)$ plane by a general amount

$$\begin{pmatrix} v \\ w \end{pmatrix} \longrightarrow \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} v \\ w \end{pmatrix} \tag{4.14}$$

where

$$\begin{aligned} v =&\ \ x^4 + ix^5 \\ w =&\ \ x^8 + ix^9 \ . \end{aligned} \tag{4.15}$$

The unrotated branes are in the following directions. The $NS5$-branes are stretched in the $x^\mu, v$ directions, with $\mu = 0, 1, 2, 3$, and at a point in $w$ and the $D6$-branes are stretched in the $x^\mu, x^7, w$ directions and at a point in $v$. We rotate the rightmost $NS5$-brane by an angle $\theta$ so that it now lies in the direction

$$\begin{aligned} v_\theta =&\ \ v\cos\theta + w\sin\theta \\ w_\theta =&\ \ -v\sin\theta + w\cos\theta \ . \end{aligned} \tag{4.16}$$

We label the rotated brane the $NS5_\theta$-brane. The $NS5_\theta$-brane is now at a point in $w_\theta$[1]

$$w_\theta = 0 \Rightarrow \quad w = v \tan\theta \equiv \mu(\theta) v \ . \tag{4.17}$$

We label the $NS5$-brane rotated by $\theta = \pi/2$ the $NS5'$-brane, it will play an important role in the descriptions of $\mathcal{N} = 1$ SQCD. The $NS5'$-brane is stretched in the $x^\mu, w$ directions and located at $v = 0$.

When we consider a system with fourbranes stretched on the interval between two fivebranes, in order to be able to rotate one of the $NS5$-branes all the $D4$-branes must be located at a certain point in the $(v, w)$ plane: $v = w = 0$ and the low energy effective field theory on the D4-branes approaches the origin of the Coulomb branch. The rotation of the $NS5$-brane by an angle $\theta$ gives a mass $\mu(\theta) \equiv \tan\theta$ to the chiral superfield in the adjoint representation of the gauge group $SU(N_c)$ belonging to the $\mathcal{N} = 2$ vectormultiplet which breaks supersymmetry from $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$. $\mu(\theta)$ describes fluctuations of the D4-branes along the surface of the $NS5$-branes. Now after the rotation the fluctuations along the surface of the $NS5$-branes by the D4-branes are further restricted. We can write the effect of the rotation as the superpotential

$$W \sim \mu(\theta)\Phi^2 + \lambda \sum_{i=1}^{N_f} \tilde{Q}_i \Phi Q^i \ . \tag{4.18}$$

Integrating out the massive adjoint field $\Phi$ we obtain $\mathcal{N} = 1$ SQCD with superpotential

$$W \sim \frac{\lambda^2}{\mu} \tilde{Q}_i Q^j \tilde{Q}_j Q^i \ . \tag{4.19}$$

The limit $\theta \rightarrow 0$ corresponds to $\mathcal{N} = 2$ SQCD while the limit $\theta \rightarrow \pi/2$, *i.e.* the adjoint field $\Phi$ is infinitely massive, corresponds to $\mathcal{N} = 1$ SQCD with vanishing superpotential.

Alternatively, we may rotate both of the fivebranes by the same angle. There is now an angle between the fivebranes and the sixbranes and we expect the Yukawa coupling $\lambda$ to vary with the rotation angle $\theta$. In this situation the massless adjoint field is now associated with fluctuations of the fourbranes along the $v_\theta$ direction. The positions of the $D4$-branes on the

[1]We could also rotate in the $w$ or $v$ planes causing shifts in these coordiantes.

interval between the $NS5$-branes correspond to expectation values $\langle\Phi\rangle$ and parametrise the Coulomb branch. The mass of the quarks is determined by the $4-6$ stretching between the $N_c$ D4-branes and the $N_f$ D6-branes and from the $\lambda(\theta)\langle\Phi\rangle$ term in the superpotential. These $4-6$ strings have minimal length $\langle\Phi\rangle\cos\theta$. Thus, the Yukawa coupling of the quarks depends on $\theta$ by $\lambda(\theta)=\cos\theta$ and we have the superpotential

$$W=\lambda(\theta)\sum_{i=1}^{N_f}\tilde{Q}_i\Phi Q^i\ . \tag{4.20}$$

If the $NS5$-branes remain unrotated we have the $\mathcal{N}=2$ description while $\theta=\pi/2$ is equivalent to rotating one of the NS5-branes to an $NS5'$-brane and we recover the situation of $\mathcal{N}=1$ SQCD with vanishing superpotential.

We now move on to the brane configurations of the electric and magnetic pictures of $\mathcal{N}=1$ SQCD whose field theory is covered in Sections 2.2 and 2.3. We will show how to pass from one picture to another by a series of brane transitions and thus realise Seiberg duality in the classical brane picture. This work was first covered in [10, 11] using the results of [55] who found that upon crossing one another a non-parallel D6-brane and an $NS5$-brane create a D4-brane stretched between them.

### 4.3.1 The Electric Picture of SQCD

The electric description of $\mathcal{N}=1$ SQCD with gauge group[2] $U(N_c)$, $N_f$ flavours of chiral superfields in the fundamental and antifundamental representations, and vanishing superpotential is given in terms of branes stretched in the following directions: an NS5-brane stretched in the $x^\mu, v$ directions and an NS5$'$-brane stretched in the $x^\mu, w$ directions separated by a distance $s_6$ in the $x^6$ direction with the $NS'$-brane to the right of the NS-brane in the $x^6$ direction. Stretch between them, in the $x^6$ direction, $N_c$ D4-branes extended in the $x^{\mu,6}$ directions. Add to this configuration $N_f$ $D6$-branes stretched in the $x^\mu, w$ directions to the left of the $NS5$-brane, where each D6-brane is

[2]Quantum mechanically, the $U(1)$ factor will decouple and sometimes we will just consider the case of $SU(N_c)$.

connected to the $NS5$-brane by a single $D4$-brane. The brane configuration is shown in Figure 4.1

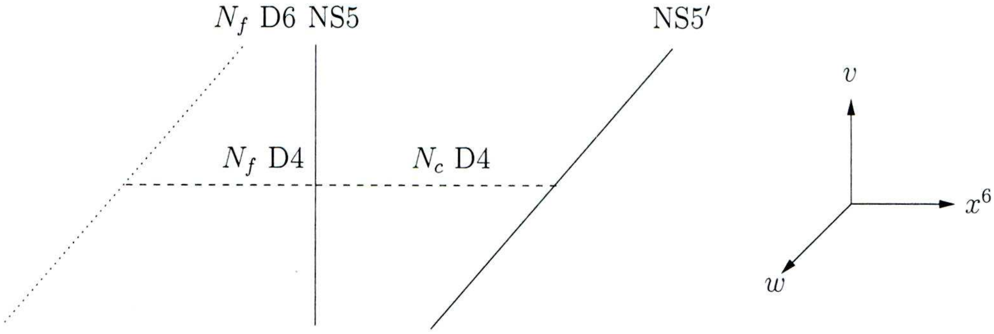


Figure 4.1: The electric theory with $N_f$ flavours of massless quarks

We will now look at more closely how the brane configuration is related to the gauge theory.

**Moduli Space of Vacua**

When $N_f < N_c$ we can break the gauge group $U(N_c) \rightarrow U(N_c - N_f)$. The complex dimension of the moduli space of vacua is $2N_cN_f - (N_c^2 - (N_c - N_f)^2) = N_f^2$. For $N_f \geq N_c$ we can completely break the gauge symmetry and the complex dimension of the moduli space is $2N_cN_f - N_c^2$. To break the gauge group in the brane configuration we must enter the Higgs phase. We split some or all of the $N_c$ D4-branes on the $N_f$ D6-branes. For $N_f \geq N_c$ the complete breaking of the gauge group is described as follows. Break the first D4-brane into $N_f + 1$ pieces connecting the NS and NS′-branes via the D6-branes with the first segment connecting the NS-brane to the first (left-most) D6-brane, the second connecting the first and second D6-brane and so forth. The final segment connects the $N_f$th (right-most) D6-brane to the $NS'$. The second D4-brane is broken is broken into $N_f$ pieces in a similar fashion, with the first segment now connecting the NS-brane to the second D6-brane (because of the s-rule) and all other segments arranged as before. Similarly the third D4-brane is broken into $N_f - 1$ segments with the first connecting the NS-brane to the third D6-brane *etc.* The final ($N_c$th) D4-brane is broken into $N_f + 1 - N_c$ pieces with the first segment connecting the NS-brane to the

$N_c$th D6-brane, the second connecting the $N_c$th D6-brane to the $N_c + 1$th, *etc.*

Now we wish to count the dimension of the moduli space for the brane configuration. A $D4$-brane stretched between two $D6$-branes has two complex massless degrees of freedom and a $D4$-brane stretched between a $D6$-brane and an $NS5'$-brane has one complex massless degree of freedom. Thus, we can conclude that the dimension of the moduli space is given by

$$N_f \geq N_c : \qquad \dim\mathcal{M}_H = \sum_{l=1}^{N_c} [2(N_f - l) + 1] = 2N_f N_c - N_c^2 \ , \qquad (4.21)$$

which agrees with the field theory result.

**Mass Deformations**

On the field theory side we can add mass for the quarks via the superpotential

$$W = -m_i^j Q^i \tilde{Q}_j \ , \qquad (4.22)$$

where $m$ is a non-degenerate $N_f \times N_f$ mass matrix. To give mass to the quarks in the brane picture we displace the $D6$-branes relative to the $D4$-branes[3] in the $v$ direction, see Figure 4.2. This leads to the same superpotential as in the gauge theory with $m$ constrained by

$$[m, m^\dagger] = 0 \ , \qquad (4.23)$$

and the locations of the D4-branes are given by the eigenvalues [4] of $m$.

**Global Symmetries**

$\mathcal{N} = 1$ SQCD with gauge group $SU(N_c)$, $N_f$ flavours of quarks in the fundamental and anti-fundamental representations of the gauge group and with vanishing superpotential has the non-anomalous global symmetry

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_R \ . \qquad (4.24)$$

[3]We could also displace the $D6$-branes relative to the $NS5'$-brane.

[4]This property is inherited from the underlying $\mathcal{N} = 2$ theory.

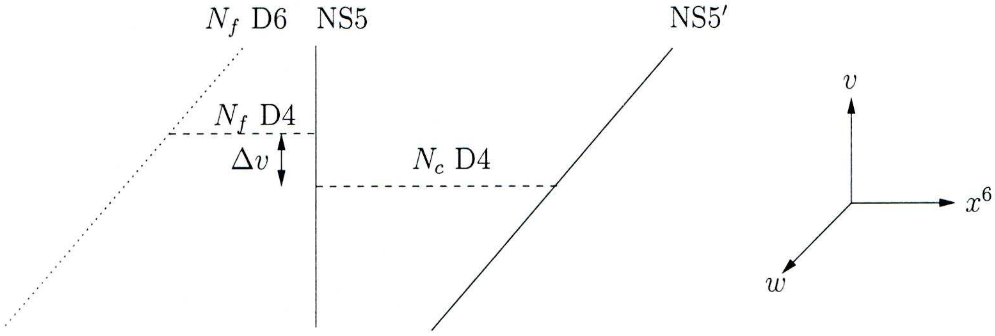


Figure 4.2: The electric theory with $N_f$ flavours of massive quarks

In the brane configuration we only expect to see the classical global symmetry

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_a \times U(1)_x \ . \tag{4.25}$$

We now examine the global symmetry of the low energy gauge theory of the brane configuration. The gauge group on the D4-branes is $U(N_c)$, quantum effects decouple the $U(1)$ (corresponding to the baryon number) part of this so we are left with the gauge group $SU(N_c)$. We have an $SU(N_f)$ gauge symmetry on the $N_f$ D6-branes, this is a global symmetry on the D4-branes. The other $SU(N_f)$ symmetry is realised in the infrared limit when the D6-branes are at the same point as the NS-branes.

Rotations in the $v$ and $w$ directions give rise to the $U(1)_x$ and $U(1)_a$ symmetries. These rotations are $R$-symmetries, the mass parameters are charged under $U(1)_{45}$, while the quarks $Q$, $\tilde{Q}$ are charged under $U(1)_{89}$.

Thus, at least heuristically, we see that the brane configuration describes the dynamics of the electric picture of $\mathcal{N} = 1$ SQCD. We will now turn to the magnetic picture.

### 4.3.2 The Magnetic Picture of SQCD

Assume $N_f > N_c$ then the magnetic description of $\mathcal{N} = 1$ SQCD with gauge group $U(N_f - N_c)$, $N_f$ fundamental flavours of quarks $q$, $\tilde{q}$, magnetic meson

$M$ and superpotential

$$W_{mag} = \text{Tr } Mq\tilde{q} \ , \tag{4.26}$$

is obtained as the low energy limit of the following brane configuration. Stretch $N_f - N_c$ $D4$-branes between an $NS5'$-brane and an $NS5$-brane, where $x^6(NS5') < x^6(NS5)$. There are also $N_f$ D6-branes to the left (with respect to $x^6$) of the NS5′-brane, connected to the NS5′-brane by $N_f$ $D4$-branes. The brane configuration is shown in Figure 4.3 and the branes are extended as in the electric description. The gauge bosons are identified as coming from $4-4$ strings connecting different colour fourbranes, while the $N_f$ quarks come from $4-4$ strings connecting the $N_c$ colour fourbranes with the $N_f$ flavour fourbranes. The magnetic meson is identified with $4-4$ strings connecting different flavour fourbranes. The coupling of the open strings implies the superpotential (4.26). The deformations of the brane configuration are re-

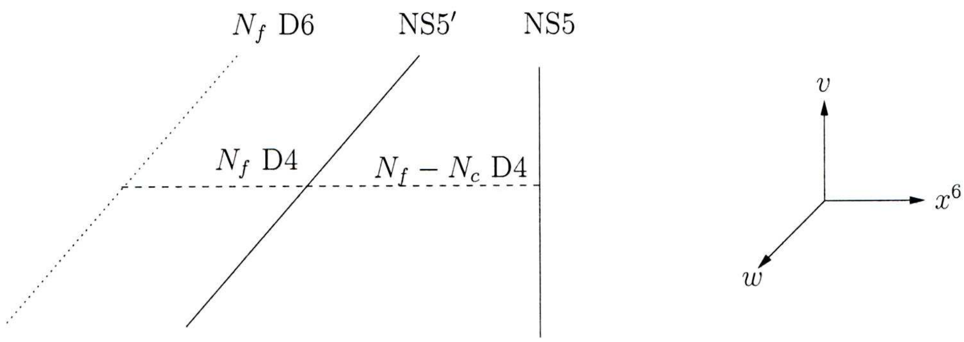


Figure 4.3: The magnetic theory with $N_f$ massless flavours of quarks

lated to the gauge theory analysis in a very similar way to that of the electric picture described above except that this time there is the additional gauge singlet magnetic meson $M$ and superpotential (4.26). We will look at adding mass deformations for the quarks and for the magnetic meson.

### Mass deformations for the quarks

We can give mass to the quarks by deforming the superpotential to

$$W_{mag} = \text{Tr } Mq\tilde{q} + m\text{Tr } q\tilde{q} \ . \tag{4.27}$$

We can absorb the term containing $m$ in the superpotential into an expectation value for the field $M$: $\langle M\rangle$ which can then be used to describe the moduli space of vacua.

In the brane configuration this is described by splitting the $N_f$ flavour D4-branes on the $D6$-branes in the same manner as when entering the Higgs phase. This results in a total of $N_f^2$ massless modes corresponding to the $N_f^2$ components of $M$. $N_f$ of the massless modes are described by fluctuations of D4-branes stretched between the $NS5'$-brane and the right-most $D6$-brane in the $w$ direction, and the $\sum_{l=1}^{N_f-1} 2l = N_f(N_f-1)$ massless modes are identified with the fluctuations of the D4-branes connecting different D6-branes in the $x^{6,7}, w$ directions.

### Adding a linear term for M

To add a linear term for $M$ we deform the superpotential (4.26) to

$$W_{mag} = \text{Tr } M(q\tilde{q} - m) \tag{4.28}$$

After integrating out the massive field $M$ the gauge group is completely broken and we may associate the eigenvalues of $m$ with the Higgs expectation values.

In terms of the brane picture, these deformations correspond to aligning the $N_f - N_c$ colour D4-branes with the $N_f$ flavour D4-branes and then connecting them so that they now stretch between the $NS5$-brane and a $D6$-brane. We can then move the connected D4-branes in the $v$ plane. If $m$ has rank $n(\leq N_f - N_c)$ then we can connect $n$ of the fourbranes. We can move these connected D4-branes in the $v$ plane again and this breaks the gauge group $U(N_f - N_c) \to U(N_f - N_c - n)$.

### Adding a quadratic term for M

To add a quadratic term for $M$ we deform the superpotential (4.26) to

$$W_{mag} = \text{Tr } Mq\tilde{q} + \frac{\alpha}{2}M^2 \ . \tag{4.29}$$

In the brane configuration this corresponds to rotations of the D6 and $NS5'$-branes in the $(v, w)$ plane. We rotate the branes by an angle $\theta$ so that they lie in the $w_\theta$ direction with

$$w_\theta = v \sin\theta + w \cos\theta \ . \tag{4.30}$$

The angle of rotation is related to the superpotential by

$$\alpha\Lambda = \tan\theta \ , \tag{4.31}$$

where $\Lambda$ is the scale of the gauge theory.

### 4.3.3 Seiberg's Duality In The Classical Brane Picture

The electric gauge theory with gauge group $U(N_c)$ and magnetic gauge theory with gauge group $U(N_f - N_c)$ described above are equivalent in the infrared via a Seiberg duality [1,2]. We have constructed the brane configurations of the electric and magnetic theories and we will now show how to move from one brane configuration to the other thus realising Seiberg duality in the classical brane picture [10,11].

We assume $N_f > N_c$ and start with the brane configuration of the electric theory. We want to deform it in order to get to the magnetic theory. Firstly, we enter the Higgs phase of the electric theory by connecting the $N_c$ colour D4-branes to $N_c$ of the $N_f$ flavour D4-branes. After doing this procedure reconnect the resulting D4-branes, obeying all the rules of brane configurations in the most general way possible. The resulting moduli space is $2N_fN_c - N_c^2$ dimensional and there are now $N_f - N_c$ $D4$-branes stretched between the $NS5$-brane and D6-branes and $N_c$ $D4$-branes stretched between the $NS5'$-brane and $D6$-branes.

Now that we are in the Higgs phase, we can move the two NS5-branes relative to each other. We can take the $NS5$-brane around the $NS5'$-brane in the $x^7$ direction and move the NS5-brane past the NS5$'$-brane in the $x^6$ direction.

The low energy gauge theory of the resulting brane configuration is the Higgs phase of a different gauge theory. We move to the root of this Higgs branch by aligning the $N_f - N_c$ $D4$-branes stretched between the $NS5$-brane and D6-branes, with the $N_c$ D4-branes stretched between the $NS5'$-brane and the D6-branes.

We then reconnect $N_f - N_c$ of the $D4$-branes and we are left with $N_f - N_c$ $D4$-branes stretched between the $NS5'$and $NS5$-branes (now to the right in $x^6$ of the $NS5'$-brane) and $N_f$ $D4$-branes stretching between the NS5′-brane and the $N_f$ D6-branes (which are to the left of the $NS'$-brane). This is exactly the brane configuration of the magnetic theory and we have realised Seiberg duality in brane configurations.

We can smoothly vary from the electric to the magnetic brane configurations by varying the scale $\Lambda$ (related to the distance in $x^6$ between the NS5 and $NS5'$-branes).

We can map the gauge theory deformations between the electric and magnetic theories in terms of brane configurations. Turning on masses for the quarks in the electric theory corresponds to moving the $D6$-branes away from the $D4$-branes (or equivalently from the $NS5'$-brane) in the $v$ plane. In the magnetic theory, this corresponds to giving Higgs expectation values to the magnetic quarks.

If we turn on expectation values for the electric quarks by breaking the stack of $N_f$ D4-branes on the $N_f$ D6-branes, in the magnetic theory this corresponds to varying the expectation value of the magnetic meson $M$ and thus giving masses to the magnetic quarks.

## 4.4 Brane Configurations of Metastable Field Theories

We examine the brane configurations reproducing the field theory of [15] considered in [19–21], and of [17] considered in [23] .

## 4.5 The Brane Configuration of the ISS Model

Start with the electric picture of $\mathcal{N} = 1$ SQCD with gauge group $SU(N_c)$, $N_f$ massive fundamental flavours[5] of quarks with superpotential

$$W_{el} = m\mathrm{Tr}\ Q\tilde{Q}\ . \tag{4.32}$$

The brane configuration giving this low-energy gauge theory is shown in Figure 4.2. The quarks have been made massive by moving the D6-branes in the $v$ direction by a distance $\Delta v$. The coefficient in the superpotential $m$ is related to $\Delta v$ by the equation

$$m = \frac{\Delta v}{l_s^2}\ . \tag{4.33}$$

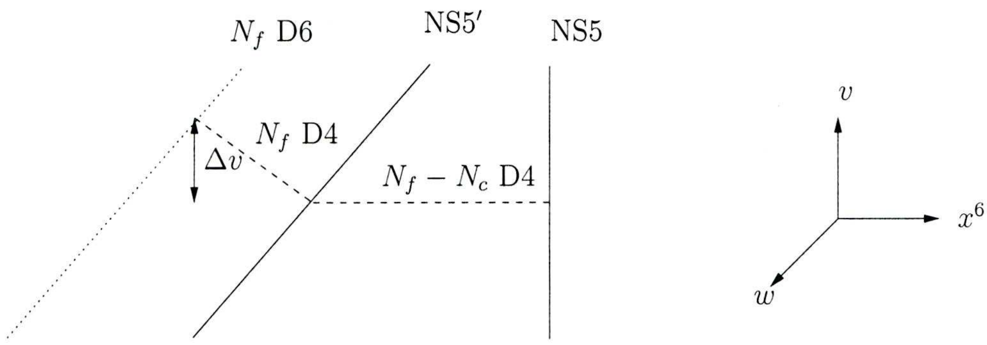


Figure 4.4: The non-supersymmetric deformed magnetic theory

Now consider making the same movement of D6-branes in the magnetic theory with gauge group $SU(N_f - N_c)$, $N_f$ flavours of massless quarks and superpotential

$$W_{mag} = \mathrm{Tr}\ Mq\tilde{q} \tag{4.34}$$

shown in Figure 4.3. When the D6-branes are moved off in the $v$ direction, the $N_f - N_c$ colour D4-branes move along with the D6-branes in order to preserve supersymmetry. However, the $N_f$ flavour D4-branes are not free to move in

[5]We consider the case of equal quark masses. The case of unequal quark masses is similar.

the $v$ direction. This causes a misalignment of branes shown in Figure 4.4 and thus, a breaking of supersymmetry. The SUSY-breaking can be interpreted as the rank condition caused by the $F$-term of the magnetic meson as in the field theory case discussed in Section 3.1. As in the field theory, this $F$-term can be partially cancelled. To do this we connect $N_f - N_c$ of the misaligned D4-branes to the $N_f - N_c$ colour D4-branes. We are left with the configuration of Figure 4.5 where there are $N_f - N_c$ D4-branes connecting the NS5 and D6-branes, and $N_c$ D4-branes between the NS5$'$ and D6-branes. These $N_c$ D4-branes are free to move in the $w$ direction reproducing the eigenvalues of the pseudomoduli of [15].

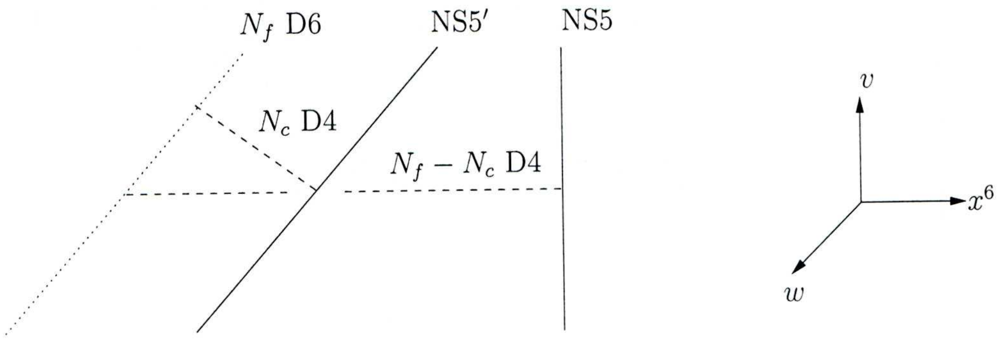


Figure 4.5: The minimum energy non-supersymmetric magnetic theory

### 4.5.1 The Electric Theory

We can perform a Seiberg duality on the brane configuration of the SUSY-breaking minimum of the magnetic theory to get to the non-SUSY electric configuration. To do this we interchange the positions of the NS and NS$'$-branes of the susy-breaking minimum magnetic brane configuration to obtain Figure 4.6.

### 4.5.2 Estimating the Lifetime

Unlike in the field theory description of the ISS model, we cannot estimate the lifetime of the metastable vacua from the brane configuration. We can qual-

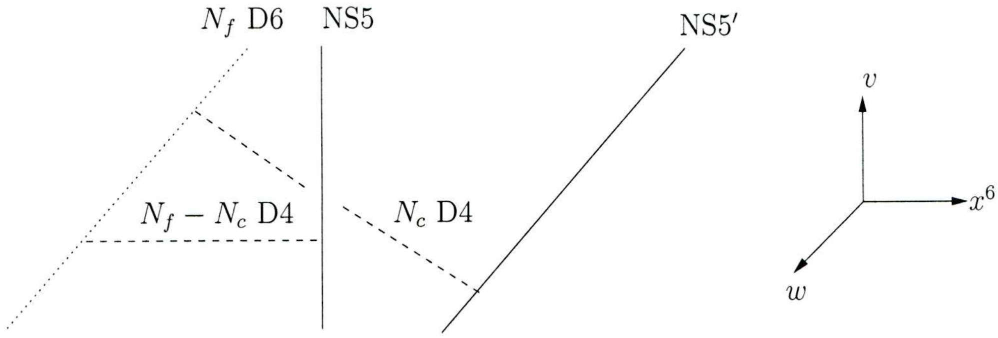


Figure 4.6: The non-supersymmetric electric theory

itatively explain the lifetime by noting that the potential energy decreases when going from the non-supersymmetric configuration to the supersymmetric configuration with massive quarks. This is due to an initial increase in total length of the D4-branes, and therefore the energy of the system, followed by a decrease in length (energy) when we move from the nonsupersymmetric configuration to the supersymmetric configuration.

## 4.6 The Brane Configuration of the Giveon-Kutasov Metastable Model

We now describe the brane configuration [23] of the metastable model of Giveon and Kutasov [17] covered in Section 3.3. Start with the brane configuration of Figure 4.3 describing the magnetic picture of $\mathcal{N} = 1$ SQCD with gauge group $SU(N_f - N_c)$, scale $\Lambda$, $N_f$ fundamental flavours of quarks $q$, $\tilde{q}$, magnetic meson $M$ and superpotential

$$W_{mag} = \frac{1}{\Lambda} \mathrm{Tr}\ M q \tilde{q}\ . \tag{4.35}$$

We wish to make two deformations to the brane configuration which were covered in section 4.3.2. First we displace the D6-branes in the $v$ direction relative to the $NS'$-brane, this has the effect of adding a linear term for $M$

to the superpotential:

$$W_{mag} = \frac{1}{\Lambda}\text{Tr } Mq\tilde{q} - m\text{Tr } M \ . \tag{4.36}$$

The second deformation we make is rotating the D6-branes and $NS'$-brane in the $(v, w)$ plane by an angle

$$w_\theta = v \sin\theta + w \cos\theta \ . \tag{4.37}$$

This has the effect of adding a quadratic term for $M$ to the superpotential:

$$W_{mag} = \frac{1}{\Lambda}\text{Tr } Mq\tilde{q} + \frac{\alpha}{2}M^2 \ . \tag{4.38}$$

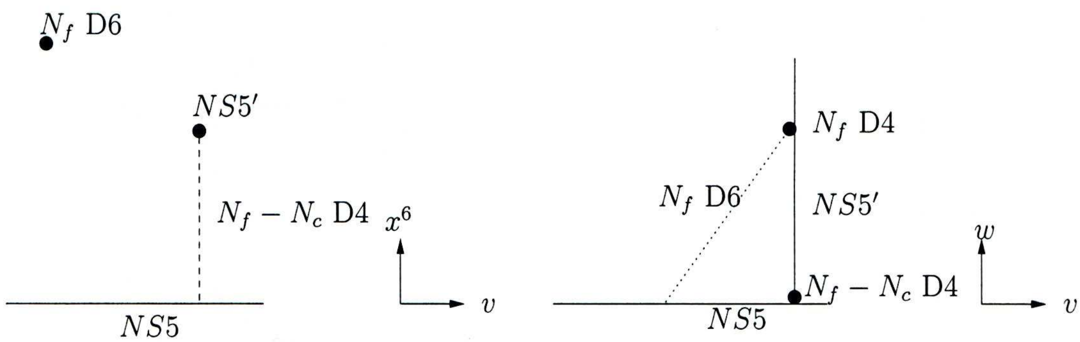


Figure 4.7: The brane configurations describing the deformed magnetic theory in slices of the $w = 0$ and $x^6$ planes

If we turn on both deformations we are left with the brane configuration of Figure 4.7 with superpotential given by

$$W_{mag} = \frac{1}{\Lambda}\text{Tr } Mq\tilde{q} - m\text{Tr } M + \frac{\alpha}{2}M^2 \ . \tag{4.39}$$

This gives exactly the field theory of [17] discussed in Section 3.3 with the supersymmetric vacuum at

$$M = \begin{pmatrix} 0 & 0 \\ 0 & \frac{m}{\alpha}\mathbb{I}_{N_f-k} \end{pmatrix} , \tag{4.40}$$

$$\tilde{q}q = \begin{pmatrix} m\Lambda \mathbb{I}_k & 0 \\ 0 & 0 \end{pmatrix}, \tag{4.41}$$

where the classical supersymmetric vacua are labelled by $k = 0, 1 \ldots, N_f - N_c$.

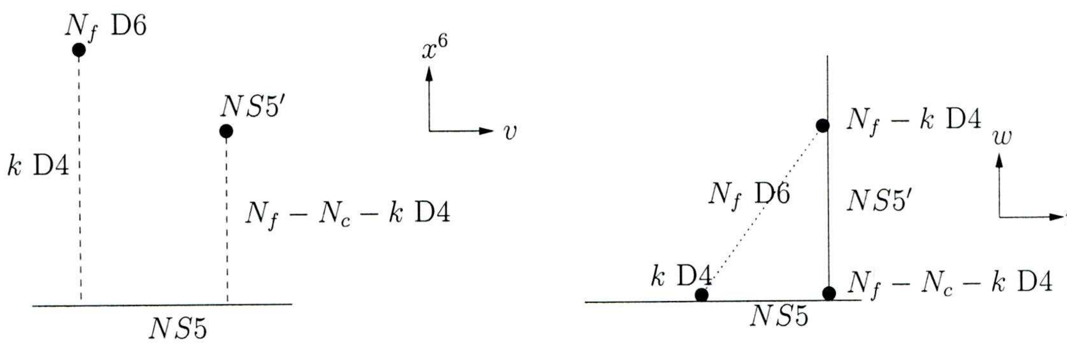


Figure 4.8: The brane configurations describing the supersymmetric vacua in slices of the $w = 0$ and $x^6$ planes

For an arbitrary value of $k$, the gauge group is broken by the expectation values of the quarks $SU(N_f - N_c) \to SU(N_f - N_c - k)$. The brane configuration of Figure 4.7 corresponds to the case $k = 0$. To describe the case of arbitrary $k$, we must connect $k$ of the flavour D4-branes to $k$ of the colour D4-branes and this configuration is shown in Figure 4.8. There is some movement of the branes in order to minimise the energy of the system, the $k$ D4-branes move to a new point in $(v, w) = (v_2, 0)$.

The $N_f - N_c - k$ D4-branes stretched on the interval between the $NS5$ and $NS5'$-branes correspond to the gauge group $SU(N_f - N_c - k)$, while the positions of the $N_f - k$ D4-branes stretched between the D6-branes and $NS'$-brane in the $w$ direction correspond to the expectation value of $M$ (4.40).

### 4.6.1 The Electric Theory

We now wish to describe the electric brane configuration of [23].

We start with the electric description of Figure 4.1 describing $\mathcal{N} = 1$ SQCD with gauge group $SU(N_c)$, $N_f$ massless flavours of quarks $Q$, $\tilde{Q}$ and vanishing superpotential. We make the same deformations as in the magnetic

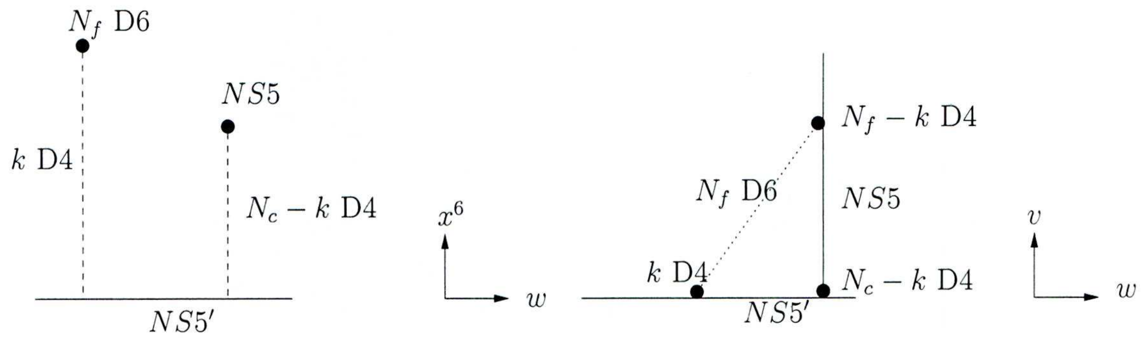


Figure 4.9: The brane configurations describing the supersymmetric vacua of the electric theory in slices of the $v = 0$ and $x^6$ planes

theory: move the D6-branes in the $v$ direction by $v_2$ and rotate them by the angle $\theta$. We obtain the configuration of figure 4.9. These deformations correspond to adding the superpotential

$$W_{el} = \frac{\alpha}{2}\text{Tr } (\tilde{Q}Q)^2 - m\text{Tr } \tilde{Q}Q \ , \tag{4.42}$$

which describes the classical supersymmetric vacua of [17] which are labelled by $k = 0, 1, \dots N_c$.

### 4.6.2 The Brane Configuration of the Metastable Vacua

In brane dynamics the one-loop effects governed by the Coleman-Weinberg potential are described by the classical gravitational attraction of the D4-branes to the $NS5$-branes [23, 27]. The brane description of the metastable vacua of [17] was shown in [23] using these gravitational effects, see Figure 4.10. The $n$ light flavours of the gauge theory correspond to 4−4 fundamental strings stretched between the stack of $n$ flavour D4-branes and the stack of $N_f - N_c - k$ colour D4-branes.

We can arrive at this configuration by starting from the configuration of Figure 4.8 in the following way. Move $n$ of the $N_f - k$ D4-branes stretched between the $NS5'$ and D6-branes towards the $NS5$-brane. The brane configuration describing the metastable vacua is locally stable but it can decay

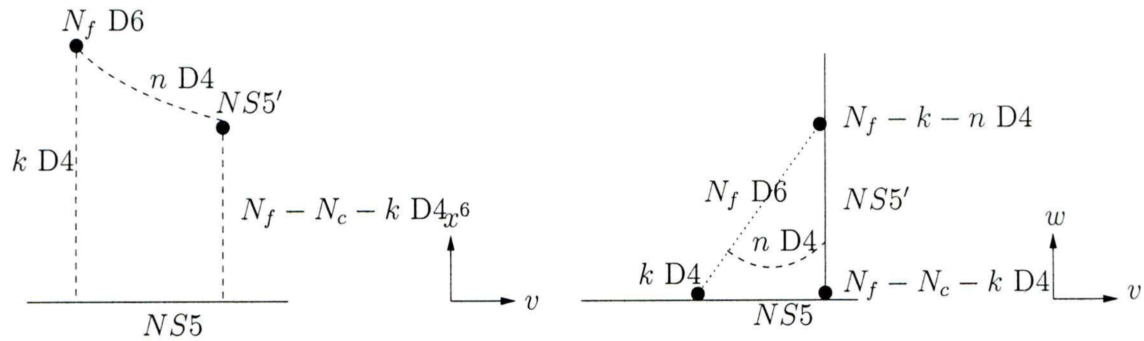


Figure 4.10: The brane configurations describing the non-supersymmetric vacua in slices of the $w$ and $x^6$ planes

to the brane configuration describing the supersymmetric vacua in two ways.

The first way is if $N_f - N_c - k > 0$ then the end points of the stack of $n$ D4-branes on the $NS5'$-brane can connect to the stack of $N_f - N_c - k$ D4-branes so that the connected branes are now on the intersection of D6 and $NS$-branes. The second way sees the stack of $n$ D4-branes moving in the $w$ direction to the configuration of Figure 4.7.

### 4.6.3 Estimating the Lifetime

The processes describing the decay from the brane configuration of the non-supersymmetric vacua to the brane configuration of the supersymmetric vacua involves an initial increase in energy (length of the branes) and then a decrease. This qualitatively explains the long lifetime of the metastable vacua.

## 4.7 T-Duality

We can see T-duality in string perturbation theory [49–53], it takes a weakly coupled string vacuum to another weakly coupled string vacuum. If we look at IIA string theory in 8+1 dimensions and have the $x^i$ direction compactified on a circle of radius $R_i$ then at large $R_i$ the theory becomes $9+1$ dimensional while at small $R_i$ it remains $8+1$ dimensional. This is however a naive picture as strings can wind round the circle in the $x^i$ direction. These strings have energy $\frac{nR_i}{l_s^2}$ where $n$ is the winding number, and become light in the limit $R_i \to 0$ producing a continuous Kaluza-Klein spectrum and making the theory $9+1$ dimensional again. The theories at large and small radii are equivalent and this is the simplest form of T-duality.

Branes in IIA string theory transform as follows under the action of a T-duality $T_i$ [49–53]:

- A fundamental IIA string wound $n$ times around the circle transforms into a fundamental IIB string carrying momentum $\frac{n}{R_i}$. An unwound fundamental IIA string carrying momentum $\frac{m}{R_i}$ transforms to a fundamental IIB string wound $m$ times around the circle.
- A IIA Dirichlet $p$ brane wrapped around the circle transforms into an unwrapped IIB Dirichlet $p-1$ brane. An unwrapped IIA Dirichlet $p$ brane transforms into a Dirichlet $p+1$ brane wrapped around the circle.
- A IIA NS fivebrane wrapped around the circle transforms into a IIB NS fivebrane wrapped around the circle. An unwrapped IIA NS fivebrane transforms into the KK monopole carrying magnetic charge under $G_{\mu,i}$.

### 4.7.1 IIB Calabi-Yau to IIA Brane Configurations

We now show how to realise the IIA brane configurations of this chapter from wrapped branes in IIB via a T-duality. An $A$-type singularity,

$$x^2 + y^2 = \prod_i (z - a_i) \tag{4.43}$$

is related by T-duality to a configuration of parallel NS fivebranes [56]. This can be shown by performing the T-duality on the elliptic fibre along the natural $S^1$ action on $(x, y)$. In the original geometry (4.43), the elliptic fibre undergoes a monodromy transformation $\tau \to \tau + 1$ around each point $z = a_i$ where $\tau$ is the complex structure [56]. After the T-duality which exchanges IIA and IIB, this becomes a unit integral shift in the NS-NS $B$-field on the fibre. Therefore the integral of $H = dB$ on a small circle around $z = a_i$ times the fibre gives 1, namely the region near $a_i$ carries the minimum unit of the NS-NS charge. This shows that the T-duality replaces the degeneration of the fibre at each $a_i$ by one NS5-brane.

We can also perform the T-duality on a collection of $A$-type singularities:

$$\begin{aligned} x^2 + y^2 &= \prod_i (z - a_i) \\ x'^2 + y'^2 &= \prod_i (z - b_i) \ , \end{aligned} \tag{4.44}$$

which gives rise to two NS5-branes with different orientation. We choose coordinates so that the NS5-branes of the first type are stretched in the $x^{0,\ldots,3,4,5}$ directions and the NS5-branes of the second type are stretched in the $x^{0,\ldots,3,8,9}$ directions. Since $x^6, x^7$ are common transverse directions to both types of NS5-branes, $(x^6, x^7)$ can be regarded as real and imaginary parts of $z$ in (4.44). Therefore the NS5-branes of the first type are located at $x^6 + ix^7 = b_i$, and the second type are located at $x^6 + ix^7 = a_i$. This is IIA brane configuration with $\mathcal{N} = 2$ supersymmetry in four dimensions of the type studied in Section 4.2.

Now consider D5-branes wrapping $S^2$ cycles $[a_i, b_j]$, locally these D5-branes look like $S^1 \times S^1$ on the fibre times the line segments on the base $z$-plane so a T-duality on the fibre squeezes the $S^1 \times S^1$ directions on the branes and leaves them stretched on the line segments on the base. Thus the D5-branes turn into parallel (in order to preserve supersymmetry) D4-branes connecting the NS5-branes. We choose coordinates so that this direction is parallel to the $x^6$ axis and in this way we can realise the IIA brane configurations of this chapter.

# Chapter 5

# Geometrical Picture

## 5.1 $\mathcal{N} = 2$ ADE Quiver Theories

We can relate $\mathcal{N} = 2$ supersymmetric gauge theories with gauge group $\prod_{i=1}^{n} U(N_i)$ and bifundamental hypermultiplets $Q_{i,j}$ in the $(\mathbf{N_i}, \bar{\mathbf{N_j}})$ representation of $U(N_i) \times U(N_j)$ to the Dynkin diagrams of the $A_n, D_n, E_n$ simple complex Lie algebras. The $U(N_i)$ factors are identified with the vertices $v_i$ of the Dynkin diagram and the $Q_{i,j}$ are identified with the edges running from $v_i$ to $v_j$.

A-D-E singularities come from the two dimensional quotient singularity $(\mathbb{C}^2/G, 0)$ by a finite subgroup $G \subset SU(2)$. Embedded as hypersufaces $f(x, y, u) = 0$ in $\mathbb{C}^3$ they are given by

$$
\begin{aligned}
A_n : &\quad f = xy + u^{n+1} \ , \\
D_n : &\quad f = x^2 + y^2 u + u^{n-1} \ , \\
E_6 : &\quad f = x^2 + y^4 + u^3 \ , \\
E_7 : &\quad f = x^2 + uy^3 + u^3 \ , \\
E_8 : &\quad f = x^2 + y^5 + u^3 \ .
\end{aligned}
\tag{5.1}
$$

A-D-E singularities can be blown up to smooth asymptotically locally Euclidean (ALE) spaces where the singular point is replaced by a configuration of rational curves $\mathbf{P}^1$ ($\mathbf{P}^1$ is isomorphic to the Riemann sphere) for a detailed

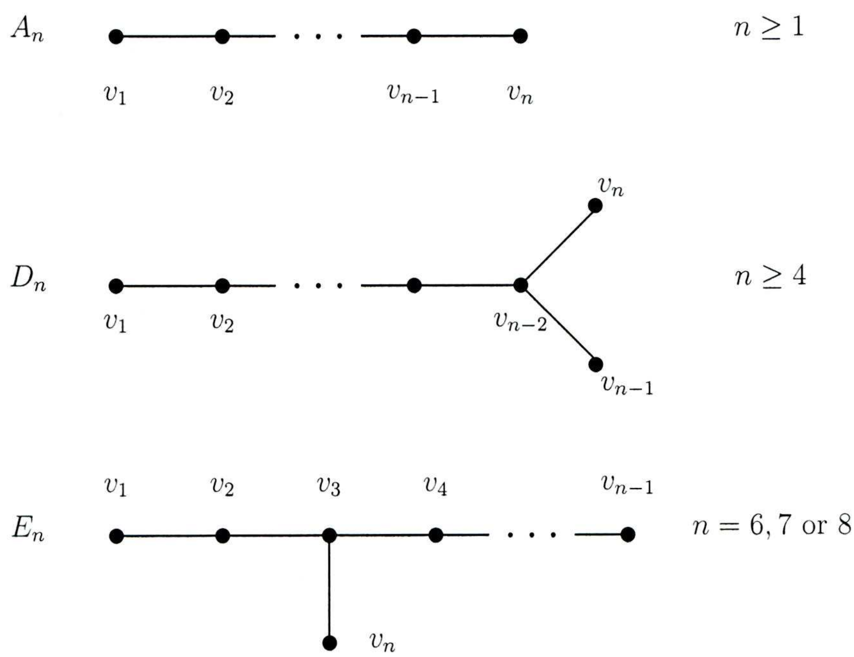


Figure 5.1: A-D-E Dynkin Diagrams and Resolution Graphs

description see [57]. The resolution graph of a degree $n$ A-D-E singularity consists of $n$ vertices each corresponding to a $\mathbf{P}^1$ cycle and edges between the vertices corresponding to when the $\mathbf{P}^1$s intersect. The A-D-E Dynkin diagram is the same as the resolution graph of the A-D-E singularities.

Consider type IIB string theory compactified on the product of a smooth ALE space and the flat complex plane $\mathbb{C}^1$. The normal bundle of each $\mathbf{P}^1$ is given by $\mathcal{O}(-2) \oplus \mathcal{O}(0)$[1]. We can wrap $N_i$ D5 branes around each of the $n$ $\mathbf{P}^1$ cycles. Macroscopically, the field theory on the D5 branes is an $\mathcal{N} = 2$ $\prod_{i=1}^{n} U(N_i)$ quiver gauge theory. The eigenvalues of the adjoint fields $\Phi_i$ of the $U(N_i)$ factors are identified with deformations along the $\mathcal{O}(0)$ sections of the normal bundle.

We now construct the resolved ALE space for the A-D-E singularities and, via a T-duality, find the corresponding IIA brane configuration. The resolutions of the A-D-E singularities can be obtained by plumbing $n$ $\mathcal{O}(-2)$ over the $\mathbf{P}^1_i$. Introduce two $\mathbb{C}^2$ for each $\mathcal{O}(-2)$ of $\mathbf{P}^1$, which we denote by $\mathbb{C}^2_{i,0}$

[1]Where $\mathcal{O}(-n)$ is the line bundle with $n$th first chern class.

and $\mathbb{C}^2_{i,\infty}$. Denote the coordinates of $\mathbb{C}^2_{i,0}$ by $Z_i$, $Y_i$ and of $\mathbb{C}^2_{i,\infty}$ by $Z'_i$, $Y'_i$. The total space of the normal bundle $\mathcal{O}(-2)$ over the $i$-th $\mathbf{P}^1_i$ is given by gluing $\mathbb{C}^2_{i,0}$ and $\mathbb{C}^2_{i,\infty}$ with the identification

$$Z'_i = 1/Z_i, \quad Y'_i = Y_i Z_i^2, \quad X' = X \ . \tag{5.2}$$

The $\mathbf{P}^1_i$ are labelled by arbitrary $X$ and are at $Y_i = Y_i' = 0$. We glue together the zero sections $\mathcal{O}(0)$ of neighbouring $\mathbf{P}^1$ cycles by identifying the corresponding North poles of the $\mathbf{P}^1_i$ ($Z'_i = X'_i = Y'_i$) and the South poles of the $\mathbf{P}^1_{i+1}$ ($Z_{i+1} = X_{i+1} = Y_{i+1}$). The fibre (respectively base) coordinate $Y'_i$ (respectively $Z'_i$) over the $\mathbf{P}^1_i$ is exchanged with the base (respectively fibre) coordinate $Z_{i+1}$ (respectively $Y_{i+1}$) of the $\mathbf{P}^1_{i+1}$. For resolved A-D-E singularities the $\mathbf{P}^1$s are glued following the corresponding Dynkin diagram.

To find the corresponding IIA brane configuration we first consider a circle action $S_2$ on (5.2):

$$\begin{aligned}(e^{i\theta}, Z_i) &= e^{i\theta} Z_i, \quad (e^{i\theta}, Y_i) = e^{-i\theta} Y_i \ , \\ (e^{i\theta}, Z'_i) &= e^{-i\theta} Z'_i, \quad (e^{i\theta}, Y'_i) = e^{i\theta} Y'_i.\end{aligned} \tag{5.3}$$

Then this action is compatible with the plumbing since the plumbing exchanges $Z'_i$ with $Y_{i+1}$ and $Y'_i$ with $Z_{i+1}$. Since the orbits of the action degenerate along $Z_i = Y_i = 0$ and $Z'_i = Y'_i = 0$, after a T-duality we will have two parallel NS branes along the $X_i$ direction at $Z_i = Y_i = 0$ and $Z'_i = Y'_i = 0$ [26]. The total brane configuration will consist of $(n+1)$ parallel NS branes labelled from 0 to $n$. The NS branes are all parallel since $X$ is the coordinate of the trivial bundle $\mathcal{O}(0)$ over $\mathbf{P}^1$. Thus the T-dual of $N_i$ D5 branes wrapping $\mathbf{P}^1_i$ of the resolution of $A_n$ singularity will be a brane configuration of $n+1$ parallel fivebranes with $N_i$ D4 branes between the $(i-1)$-th and $i$-th NS branes as shown in Figure 5.2. The D4 branes can freely move along the direction of the NS branes corresponding to the Coulomb branch of the gauge theory. This method also works for other A-D-E singularities.

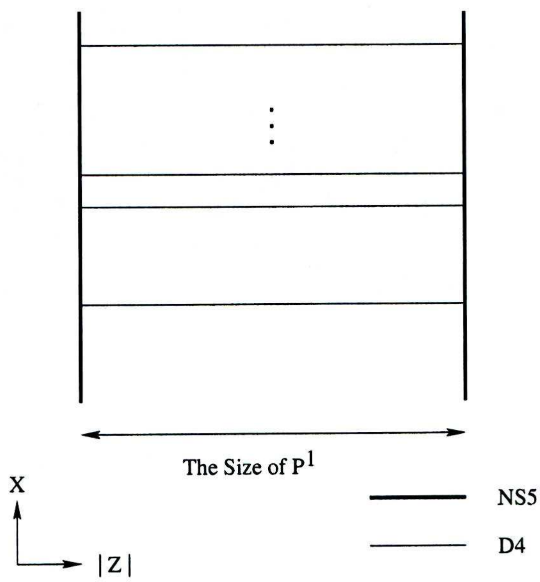


Figure 5.2: The T-dual configuration of $N$ D5 branes wrapped on $\mathbb{P}^1$ cycles with normal bundle $\mathcal{O}(-2) + \mathcal{O}(0)$.

## 5.2 Geometric Transitions

We now briefly review the geometric transition first considered with topological strings in [58] and then in the context of superstrings in [24]. The transition at large $N$ has been generalised to more complicated geometries in [25, 59–62] where the blown-up geometry involves more $\mathbf{P}^1$ cycles and the deformed geometry involves more $S^3$ cycles. We begin by summarising the work of [24] and then move on to the case of a theory with superpotential for the adjoint chiral superfield first considered in [25]. The large $N$ description of systems of resolved A-D-E singularities involves replacing the $\mathbf{P}^1$ cycles of the resolved theory with finite sized $S^3$s with RR-flux through them and NS flux through the corresponding dual cycles. We can view the large $N$ description as a transition from the resolved conifold singularity to the deformed conifold singularity.

Consider IIB string theory with $N$ D5 branes wrapped on the $S^2$ of a resolved conifold singularity[2]:

$$xy - uv = 0 \ . \tag{5.4}$$

The local geometry of this non-compact Calabi-Yau threefold is a $\mathbf{P}^1$ with normal bundle $\mathcal{O}(-1)+\mathcal{O}(-1)$ obtained from two copies of $\mathbb{C}$: $(X, Y, Z)$ and $(X', Y', Z')$ with the identification

$$Z' = \frac{1}{Z}, \quad X' = XZ, \quad Y' = YZ \ . \tag{5.5}$$

The blow-down map to the singular conifold is

$$x = X = X'Z', \quad y = ZY = Y', \quad u = ZX = X', \quad v = Y = Z'Y' \ . \tag{5.6}$$

At low energies, the gauge theory on the $N$ D5 branes is a four dimensional $\mathcal{N} = 1$ $U(N)$ pure Yang-Mills theory described by, in the small 't Hooft parameter regime (small $N$), open strings ending on the D5 branes. The fact that the string theory is compactified on a local Calabi-Yau manifold means

[2]For a review on conifolds see [63].

that there is $\mathcal{N} = 2$ supersymmetry, the presence of the D5 branes further breaks supersymmetry to $\mathcal{N} = 1$. Any deformation of the $\mathbf{P}^1$ cycle in the normal direction leads to an increase in the volume of the $\mathbf{P}^1$. Therefore, the field theory of the D5 branes contains no massless adjoint fields.

Consider the circle action on the resolved conifold

$$\begin{aligned} (e^{i\theta}, Z) \to Z, \quad (e^{i\theta}, X) \to X, \quad (e^{i\theta}, Y) \to e^{-i\theta} Y \ , \\ (e^{i\theta}, Z') \to e^{-i\theta} Z', \quad (e^{i\theta}, X') \to e^{i\theta} X', \quad (e^{i\theta}, Y') \to Y' \ . \end{aligned} \tag{5.7}$$

The orbits degenerate along the lines $Z = Y = 0$ and $Z' = Y' = 0$ which are separated by the size of the $\mathbb{P}^1$ cycle. Taking the T-dual we find two orthogonal NS branes, one in the $X$ direction and one in the $Y'$ direction.

Vafa's duality [24] states that in the large $N$ limit (the large 't Hooft parameter regime), the description above is equivalent to type IIB closed string theory on a Calabi-Yau manifold obtained after a geometric transition replacing the $\mathbf{P}^1$ with a finite sized $S^3$ *i.e.* we now have a deformed conifold singularity:

$$f = xy - uv - \mu = 0. \tag{5.8}$$

Consider a circle action on the resolved conifold

$$(e^{i\theta}, x) \to x, \quad (e^{i\theta}, y) \to y, \quad (e^{i\theta}, u) \to e^{i\theta} u, \quad (e^{i\theta}, v) \to e^{-i\theta} v \ . \tag{5.9}$$

Taking the T-dual of the deformed conifold we find an NS brane along the degeneration of the orbits described by the curve $u = v = 0$ and $xy = \mu$.

When flowing to the IR (long distance), the gauge theory on the D5 branes confines and develops a mass gap. On the string theory side, the confinement of the open string degrees of freedom can be thought of as the D5 branes disappearing. Rather than the $N$ original D5 branes, there are now $N$ units of RR flux through the $A$ cycle corresponding to the $S^3$:

$$\oint_A H = N \ , \tag{5.10}$$

where

$$H = H^{RR} + \tau H^{NS} \tag{5.11}$$

and $\tau$ is the type IIB dilaton-axion. There is also flux through the dual non-compact $B$ cycle:

$$\int_B H = -\alpha \ . \tag{5.12}$$

where $\alpha$ is the coupling constant of the gauge theory. The size of the $S^3$: $\mu$, is identified with the condensate of the $SU(N)$ glueball superfield $S = -\frac{1}{32\pi^2}\mathrm{Tr}\, \mathcal{W}_\alpha \mathcal{W}^\alpha$. The glueball $S$ is identified with the flux of the holomorphic 3-form on the compact 3-cycle of the deformed conifold

$$S = \int_A \Omega \ , \tag{5.13}$$

and the prepotential $\mathcal{F}_0$ with the integral of the holomorphic $(3,0)$ form on the non-compact 3-cycle

$$\Pi = \int_B^{\Lambda_0} \Omega = \frac{\partial}{\partial S}\mathcal{F}_0 = \frac{1}{2\pi i}(-3S\log\Lambda_0 - S + S\log S) \ , \tag{5.14}$$

where $\Lambda_0$ is a cut-off required to regulate the integral which is related to the running of the gauge coupling constant $\alpha$ [24]. The effective superpotential is given by

$$W_{\text{eff}} = \int H \wedge \Omega = \alpha S + S\log[\Lambda^{3N}/S^N] + NS \ . \tag{5.15}$$

The equation of motion for $S$ leads to the appearance of the $N$ supersymmetric vacua of $SU(N)$ pure super Yang-Mills:

$$\langle S \rangle = e^{2\pi i k/N}\Lambda^3, \qquad k = 1, \ldots, N \ , \tag{5.16}$$

this is the gluino condensation in the field theory. The gluons of $SU(N)$ get a mass so the $SU(N)$ gets a mass gap and confines. What remains is the $U(1)$ part of $U(N)$ whose coupling constant is equal to the coupling constant of the $U(N)$ theory divided by $N$.

## 5.3 Geometric Engineering of $\mathcal{N}$=1 Theories with Adjoint Field $\Phi$ and Superpotential $W(\Phi)$

We now consider a more complicated case of Vafa's geometric transition where we have a general superpotential for the adjoint field.

The simplest case of deforming the $A_n$ singularity (see Section 5.2) corresponds to adding a tree-level superpotential for the adjoint chiral superfield $\Phi$:

$$W_{tree} = \sum_{p=1}^{n+1} \frac{g_p}{p} \mathrm{Tr}\ \Phi^p \equiv \sum_{p=1}^{n+1} g_p u_p \ . \tag{5.17}$$

This breaks supersymmetry from $\mathcal{N} = 2$ to $\mathcal{N} = 1$. In the IIA brane configuration the NS brane is now curved in space compared to straight in the $\mathcal{N} = 2$ theory. The case of a geometric transition with a general superpotential for the adjoint field was considered first in [25] and we review the main results in this section.

This type of model was first studied from a field theory point of view in [45–47]. We can treat the gauge group[3] as $U(N)$. The classical gauge theory has a moduli space of vacua parametrised by the eigenvalues of $\Phi$:

$$W'(x) = \sum_{p=0}^{n} g_{p+1} x^p \equiv g_{n+1} \prod_{i=1}^{n} (x - a_i). \tag{5.18}$$

For generic values of $\Phi$ the gauge group is broken as

$$U(N) \rightarrow \prod_{i=1}^{n} U(N_i) \qquad \text{with} \qquad \sum_{i=1}^{n} N_i = N. \tag{5.19}$$

In Section 5.2 we saw how the position of the D4 branes and the Coulomb branch are related. This gives a clue as to how the geometry must be deformed; rather than having the $\mathbf{P}^1$ with coordinate $Z$ and $Z'$ at the point

[3]The gauge group may be treated as $SU(N)$ by using the $g_1$ term as a Lagrange multiplier to enforce the tracelessness of $\Phi$.

$Y = Y' = 0$, for arbitrary $X$, we should now have the $\mathbf{P}^1$ cycle at particular values of $X$: $X = a_i$ where $a_i$ are the roots of the superpotential (5.17). The deformation of the geometry is given by

$$Z' = 1/Z, \quad X' = X, \quad Y' = YZ^2 + W'(X)Z \; . \tag{5.20}$$

Note that this is only compatible with $Y = Y' = 0$ at the $n$ choices of $X = a_i$. We can wrap the $N$ D5-branes on any of choice of the vacua $a_i$; this gives a geometric realisation of the breaking of $U(N) \to \prod_i U(N_i)$ as seen in the gauge theory.

We can obtain the geometry from the small resolution of a Calabi-Yau with conifold singularities:

$$y(y - W'(x)) - uv = 0 \; . \tag{5.21}$$

By replacing the singular points with $\mathbf{P}^1$ cycles by blowing up the singularities, we can resolve the space (5.21). The resolved space is covered by two copies of $\mathbb{C}$ with coordinates $Z, X, Y$ and $Z', X', Y'$ related to each other by (5.20). The blow-down map from the singular space to the resolved space is given by

$$x = X = X', \; y = YZ = Y'Z' + W'(X'), \tag{5.22}$$

$$u = Y = Z'(Y'Z' + W'(X')), \; v = Z(YZ - W'(X)) = Y' \; . \tag{5.23}$$

The $\mathbf{P}^1$ cycles of the $\mathcal{N} = 1$ theory are fixed at the roots of $W'(x) = 0$ *i.e.* $x = a_k$, $k = 1, \ldots, n$ in contrast to the $\mathcal{N} = 2$ theory where they are at arbitrary values of $x$. The circle action $S_2$ (5.3) is compatible with the resolution (5.20). Thus, we can take a T-duality along the direction of the orbits of the action $S_2$ on (5.20). The orbits degenerate along $Z = Y = 0$ and $Z = Y' = 0$ where we will have two NS branes after a T-duality. Using the blow-down map (5.22) we see that the NS branes are in the directions $y = u = v = 0$ and $y = W'(x), u = v = 0$. Therefore, we have a system of two non-parallel NS branes and considering the full configuration with D5 branes wrapped on $\mathbf{P}^1$ cycles, the T-duality gives a IIA brane configuration

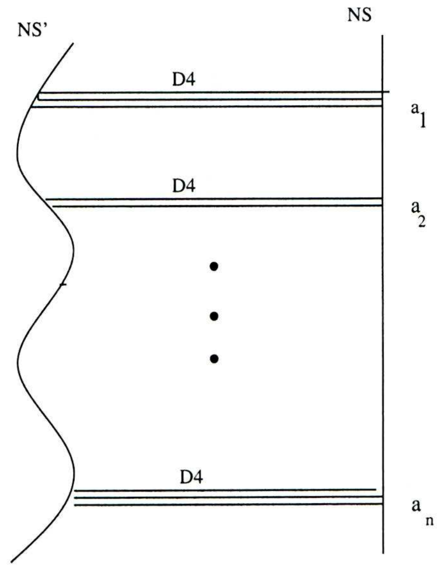


Figure 5.3: The T-dual configuration of $N$ D5 distributed among $n$ $\mathbf{P}^1$.

with two non-parallel NS branes with D4 branes stretched between them as shown in Figure 5.3.

### 5.3.1 Geometric Transition

We now obtain the large $N$ dual of the $U(N)$ theory with adjoint $\Phi$ and superpotential (5.17) by considering the geometric transition where each of the $n$ $\mathbf{P}^1$ cycles are shrunk down and replaced by finite size $S^3$s [25]. The D5 branes present before the transition are replaced by $N_i$ units of $H_R$ flux through the $i$-th $S^3$ cycle $A_i$. In addition to the Ramond-Ramond flux, there is also $H_{NS}$ flux $\alpha$ through each of the dual non-compact $B_i$ cycles, with $2\pi i\alpha = 8\pi^2/g_0^2$ given in terms of the bare coupling constant $g_0$ of the original four-dimensional $U(N)$ gauge theory. As in the simple case, the sizes of the $n$ $S^3$s correspond to the non-zero gaugino condensation expectation values of the gauge groups. Thus we have the superpotential

$$-\frac{1}{2\pi i}W_{eff} = \sum_{i=1}^{n} N_i\Pi_i + \alpha(\sum_{i=1}^{n} S_i). \tag{5.24}$$

In terms of the geometry, we have $n$ isolated conifold singular points and the complex deformation space is $n$ dimensional. We can smoothen out the geometry by adding a polynomial $f_{n-1}(x, \mu_1, ..., \mu_n)$ of degree $n-1$ in $x$ to (5.21) with $f_{n-1}(a_k, \mu_1, \ldots, \mu_n) = \mu_k$. The polynomial is given by

$$f_{n-1}(x, \mu_1, ...\mu_n) = \sum_{k=1}^{n} \mu_k \prod_{l=1, l\neq k}^{n} \frac{x - a_l}{a_k - a_l}. \tag{5.25}$$

Adding the polynomial to (5.21) we find the generalised deformed conifold

$$W'(x)y - f_{n-1}(x, \mu_1, \ldots, \mu_n) - uv = 0. \tag{5.26}$$

The $n$ three spheres $S_i^3$ of size $\mu_k$ will smooth out the singular point $x = a_k,\ y = u = v = 0$.

We now wish to find the corresponding IIA brane configuration. Consider a circle action on (5.26):

$$(e^{i\theta}, x) \rightarrow x, \quad (e^{i\theta}, y) \rightarrow y, \quad (e^{i\theta}u) \rightarrow e^{i\theta}u, \quad (e^{i\theta}v) \rightarrow e^{-i\theta}v. \tag{5.27}$$

By taking the T-dual along the orbits of the action, an NS brane will appear along $u = v = 0$ given by

$$W'(x)y = f_{n-1}(x, \mu_1, \ldots, \mu_n)\ , \tag{5.28}$$

where the orbits degenerate.

### 5.3.2 Adding Matter Fields

We now show how to add quark chiral superfields in the fundamental representation of the $U(N_i)$ gauge group factors. We add massive quarks by wrapping D5 branes on a holomorphic 2-cycle separated by a distance $m$ from the $\mathbf{P}^1$ cycle. There are two kinds of matter fields we may add: either coming from semi-infinite D4-branes or coming from D6-branes. We consider the case of semi-infinite D4-branes. These semi-infinite D4 branes give rise to $N_{k,f}$ hypermultiplets in the fundamental representation of the $U(N_k)$ factors corresponding to strings stretched between the $N_k$ D5 branes wrapping on the $\mathbf{P}^1$ cycles located at $x = a_k$, and the $N_{k,f}$ non-compact D5 branes wrapping holomorphic 2 cycles. The mass of the quarks is given by the distance between the non-compact D5 brane and the compact D5 wrapping $\mathbf{P}^1$.

The small resolution of the $\mathcal{N} = 2$ geometry is covered by two copies of $\mathbb{C}^3$ with coordinates $Z, X, Y$ (respectively $Z', X', Y'$). Semi-infinite D4 branes near the $x = a_k$ vacua attached to the NS brane are given by D5 branes wrapping non-compact holomorphic cycles at

$$Y = 0, \quad X - a_k = m_{k,1}, \ldots, m_{k,F_k} \, . \tag{5.29}$$

Semi-infinite D4 branes near $x = a_k$ attached to the NS$'$ brane are given by D5 branes wrapping non-compact holomorphic 2-cycles at

$$Y' = 0, \quad X' - a_k = n_{k,1}, \ldots, n_{k,G_k} \, . \tag{5.30}$$

## 5.4 Metastable Vacua with D5 and Anti D5 Branes

As we discussed in Section 3.4 metastable vacua can also arise in IIB string theory from configurations of branes and anti-branes wrapped on 2-cycles inside local Calabi-Yau manifolds. This type of system was first considered in [18] and we briefly review the results here.

### 5.4.1 Anti-Brane systems

As in the case of $N$ D5 branes wrapping $\mathbf{P}^1$ cycles of the resolved conifold singularity, the large $N$ dual of $N$ anti D5 branes wrapping a $\mathbf{P}^1$ cycle is also dual to the conifold deformation with flux through an $S^3$. The difference this time is that the flux through the $A$-cycle is negative. The superpotential is still given by

$$W(S) = \alpha S + \frac{1}{2\pi i} N S \left( \log \frac{S}{\Lambda_0^3} - 1 \right) \, , \tag{5.31}$$

as in the case of D5 branes but now $N$ is negative. When we consider the vacua of the theory we minimise the potential of the theory finding that there are two solutions [18]:

$$\begin{aligned} \alpha + N\tau &= 0 \\ \alpha + N\bar{\tau} &= 0\,. \end{aligned} \tag{5.32}$$

The first solution is physical when $N$ is positive *i.e.* when we have D5 branes, while the second case is physical when $N$ is negative *i.e.* when we have anti D5 branes. In terms of gaugino condensate the anti brane solution corresponds to

$$\langle S \rangle = \Lambda_0^3 \exp\left(\frac{2\pi i}{|N|}\bar{\alpha}(\Lambda_0)\right)\,. \tag{5.33}$$

The anti-brane and brane solutions preserve two different $\mathcal{N} = 1$ supersymmetries, this is made manifest by considering the underlying $\mathcal{N} = 2$ Lagrangian.

### 5.4.2 Brane-Anti Brane Systems and Geometric Metastability

We wish to construct a metastable system of branes and anti-branes. To start with we wrap D5 branes and anti-D5 branes on isolated minimal $\mathbf{P}^1$ cycles of a non-compact Calabi-Yau manifold. The local geometry of the the $\mathbf{P}^1$ cycles is of a resolved conifold singularity as reviewed in this chapter. The brane-anti brane system cannot move without an increase in energy thus giving a metastable system. If we make the vacua situated at $a_i$ very widely separated then the branes and anti-branes interact only weakly and we expect to have the usual situation on each set of branes *i.e.* an approximate $\mathcal{N} = 1$ supersymmetry with gaugino condensation at low energies. The further apart the system is the more stable it is.

### 5.4.3 Lifetime of the Vacua

The lifetime of the metastable vacua maybe estimated by using an instanton calculation [18] where the decay rate is given by

$$\Gamma \sim \exp(-S_I) \ . \tag{5.34}$$

Here $S_I$ is the instanton action of the Euclidean bounce solution which interpolates between the metastable vacuum and the true supersymmetric vacuum. The height of the potential barrier is determined in terms of the distances between the branes and anti-branes and the attraction between the branes and anti-branes is overcome by making the separation very large. The other factor in the lifetime of the vacua is the ratio between the number of branes and anti-branes which gives the difference in energies between the supersymmetric and metastable vacua. When this ratio is large then the system is more stable.

# Chapter 6

# M Theory

In this chapter we consider the M theory lift of IIA brane configurations with fourbranes and fivebranes of the type studied in Chapter 4. We start with an $\mathcal{N} = 2$, $U(N)$ brane configuration first considered in [8] and then move to $\mathcal{N} = 1$ theories considered in [9]. Finally we consider the effect of a geometric transition in the M theory picture.

## 6.1 M Theory Lift of $\mathcal{N} = 2$ Brane Configurations

First we review the M Theory Lift of $\mathcal{N} = 2$ brane configurations of [8]. Consider a system of two parallel NS branes located at $x^{7,8,9} = 0$ classically at $x^6 = 0$ and stretching in the $x^{0,\dots,5}$ directions. Put $N$ D4 branes on the interval in the $x^6$ direction between the fivebranes. The worldvolume of the fourbranes is in the $x^{0,\dots 3,6}$ directions where the extension of the fourbranes in the $x^6$ direction is finite in extent. We described this sort of system in Section 4.2. As in previous chapters we use the conventions $v = x^4 + ix^5$ and $w = x^8 + ix^9$.

Classically the end points of the fourbranes on the fivebranes have a definite position in $v$. However, when we consider the strong coupling limit we see that the ends of the fourbranes create vortexes in the worldvolume of the fivebranes and there is no definite value of $x^6$ for the endpoints [8].

To describe the system properly we eliminate this singularity by going to the strong coupling limit of IIA string theory where we find that another dimension unfolds and the theory is now described by an eleven dimensional theory- M theory.

Consider M theory on $\mathbb{R}^{10} \times \mathbf{S}^1$ where $\mathbb{R}^{10}$ is the usual flat spacetime and $\mathbf{S}^1$ is parametrised by the extra coordinate of M theory $x^{10}$ and has period $2\pi R$. A fivebrane on $\mathbb{R}^{10}$ in IIA string theory becomes an M theory fivebrane on $\mathbb{R}^{10} \times \mathbf{S}^1$ whose worldvolume is a six dimensional manifold in $\mathbb{R}^{10}$ and is located at a point in $\mathbf{S}^1$ [8]. Fourbranes on $\mathbb{R}^{10}$ in IIA string theory also become a fivebrane when lifting to M theory, this time the worldvolume is wrapped on the $\mathbf{S}^1$ [8]. In fact the D4 and NS5 branes in IIA string theory come from the same object in M theory- the M5 brane[1].

A system of $n+1$ parallel fivebranes with $N_i$ fourbranes between the $i$th and $i+1$th fivebranes becomes a single fivebrane in M theory. If the branes are extended as above the M5 brane is located at $x^{7,8,9} = 0$ and sweeps out arbitrary values of the regular spacetime coordinates $x^{0,\ldots,3}$. The remaining coordinates $x^{4,5,6,10}$ parametrise a four dimensional manifold $Q \cong \mathbb{R}^3 \times \mathbf{S}^1$. The M5 brane worldvolume is a four dimensional complex Riemann surface $\Sigma$ inside $Q$. Shrinking the extra dimension of M theory to zero by taking the limit $R \to 0$ we recover the IIA brane configuration.

### 6.1.1 The M5 Brane Solution

We parametrise the extra dimension of M theory by

$$t = e^{-s} = e^{-(x^6 + ix^{10})/R} , \tag{6.1}$$

where $s$ is not single valued due to $x^{10}$ being a periodic coordinate. We describe models by a complex curve in $Q$ in terms of the equation $F(s, v) = 0$. For fixed values of $v$ the roots of $F(t)$ give the positions of the NS5 branes in the IIA brane configuration. If we find values of $t$ for the position of the NS5 branes which are outside of the classical position of the NS5 branes then

[1]Other IIA branes may be lifted to M theory but in this work we only consider IIA four and fivebranes.

these solutions are still valid and represent the bending of the NS5 branes at large $v$. The degree of $F$ in $t$ gives the number of NS5 branes in the IIA brane configuration, while the degree of $F$ in $v$ gives the number of D4 branes in the IIA brane configuration.

Now if we consider specifically a IIA brane configuration of $n+1$ parallel NS branes where the $n_i$th NS brane is connected to the $n_{i+1}$th NS brane by $N_i$ D4 branes the low energy field theory is an $\prod_{\alpha=1}^{n} SU(N_i)$ theory with hypermultiplets in the bifundamental representation $(\mathbf{N}_i, \bar{\mathbf{N}}_{i+1})$ of $SU(N_i) \times SU(N_{i+1})$. The corresponding M5 brane is described by the equation

$$F(t,v) = t^{n+1} = k_{N_1}(v)t^n + k_{N_2}(v)t^{n-1} + \cdots + k_{N_n}(v)t + 1 \ . \tag{6.2}$$

The roots of the polynomials $k_{N_i}$ give the positions of the $N_i$ D4 branes stretching from the $n_i$th to the $n_{i+1}$th NS branes in the IIA brane configuration. We can express $k_{N_i}(v)$ as

$$k_{N_i}(v) = c_{i,0}v^{N_i} + c_{i,1}v^{N_i-1} + \cdots + c_{i,N_i}v^0 \ , \tag{6.3}$$

and we can use this equation to determine the large $v$ behaviour of the NS5 branes in the IIA picture.

## 6.2 M Theory Lift of $\mathcal{N}=1$ Brane Configurations

To break from $\mathcal{N}=2$ supersymmetry to $\mathcal{N}=1$ supersymmetry we add a mass term for the chiral superfield in the adjoint representation of the $\mathcal{N}=2$ vector multiplet. In terms of IIA brane configurations adding a mass for the adjoint field corresponds to rotating the fivebranes relative to each other. M theory considerations for $\mathcal{N}=1$ theories were first done in [9,64] where they started with an $\mathcal{N}=2$ configuration with four and fivebranes and rotated the fivebranes relative to each other to break the supersymmetry to $\mathcal{N}=1$.

We start with a IIA brane configurations with two NS branes: an NS brane located at $x^{6,7,8,9}=0$ and stretched in the $x^{0,1,2,3,4,5}$ directions and an NS$'$ brane located at $x^{4,5,7} = 0$, $x^6 = s_6$ and stretched in the $x^{0,1,2,3,8,9}$ directions. We chose the convention $s_6 > 0$ so that the NS$'$ brane is to the right of the NS brane in the $x^6$ direction. We stretch $N$ fourbranes between the two NS branes in the $x^6$ direction where the fourbranes are located at $v = w = x^7 = 0$ and are stretched in the $x^{0,1,2,3,6}$ directions.

We can describe the M5 brane of this configuration by $\mathbb{R} \times \Sigma$ where $\Sigma$ is a complex Riemann surface with coordinates $v, w, t$. To determine the form of $\Sigma$ we make use of the following: $\Sigma$ should only go to infinity at positions of the NS branes so $\Sigma$ is the punctured $v$-plane with the points $v = 0$ and $v = \infty$ removed. This behaviour can be described by the equation $w = \zeta v^{-1}$ where $\zeta$ is a complex constant.

$t$ should be described by the equation $t = F_N(v)$; if we consider fixed values of $t$ then the roots of this equation give the position of the $N$ D4 branes. We can write this condition as $F_N = \frac{K(v)}{H(v)}$ where $K(v)$, $H(v)$ are polynomials of degree $N$. The points at infinity in $x^6$ correspond to $t = 0$ and $t = \infty$, thus the only zeroes and poles of $t$ are at $v = 0$ and $w = 0$ so we may write $t$ in the form $t = v^N$. In conclusion, the curve $\Sigma$ is described by the following equations

$$w = \zeta v^{-1} \, , \quad v^N = t \, , \quad w^N = \zeta^N t^{-1} \, . \tag{6.4}$$

## 6.3 Geometric Transitions in M Theory

In this section we summarise the works [4, 5] describing the effects of a geometric transition following from the large $N$ duality between the resolved and deformed conifolds in M theory.

Recall the results of Chapter 5 where the $\mathcal{N} = 1$, $U(N)$ gauge theory with superpotential (5.17) for the adjoint superpotential and large $N$ dual were constructed in IIB string theory configurations and the corresponding IIA brane configuration was found via a T duality.

### 6.3.1 Theory with a Quadratic Superpotential for the Adjoint Field

In Section 5 we saw that the large $N$ duality of [24] can be viewed as a conifold transition. We would like to lift the configurations of Section 5.2 to M theory [4, 5] starting with the resolved conifold.

If we have $N$ D5 branes wrapping a $\mathbf{P}^1$ cycle of the resolved conifold then the corresponding IIA T dual will be a brane configuration of two fivebranes perpendicular to each other and separated in the $x^6$ direction. Here, the worldvolume of the NS brane is in the $x^{0,1,2,3,4,5}$ directions and that of the NS$'$ brane is in the $x^{0,1,2,3,8,9}$ directions. There are $N$ fourbranes stretched between the two fivebranes in the $x^6$ direction whose worldvolume is in the $x^{0,1,2,3,6}$ directions.

This configuration is lifted to a single M5 brane with worldvolume $\mathbb{R}^4 \times \Sigma$ where $\Sigma$ is a complex curve defined by

$$y = \zeta x^{-1}, \quad t = x^N \ . \tag{6.5}$$

Now we consider what happens in a conifold transition where the size of the $\mathbf{P}^1$ goes to zero. In this case $t$ then becomes a constant value on $\Sigma$ and $\Sigma$ now makes the transition from a space curve to a plane curve described by

$$t = t_0, \quad xy = \zeta e^{2\pi k/N}, \quad k = 0, 1, \ldots, N-1 \ . \tag{6.6}$$

In fact this is the M theory lift of the deformed conifold and we have realised the geometric transition in M theory. The M5 brane describes $N$ different $U(1)$ gauge theories labelled by $k$.

### 6.3.2 Theory with a General Superpotential for the Adjoint Field

The geometry and brane configurations for the $\mathcal{N}=1,\ SU(N)$ gauge theory with superpotential

$$W_{tree} = \sum_{k=1}^{n+1} \frac{g_k}{k} \mathrm{Tr}\ \Phi^k \tag{6.7}$$

were discussed in Section 5.3. We now consider the M theory lift of these configurations.

The T dual of the generalised conifold gives a IIA brane configuration with $n+1$ NS branes with $N_i$ branes stretching between the $i$th and $i+1$th NS branes. When we lift to M theory we find a single M5 brane with worldvolume $\mathbb{R}^4 \times \Sigma$ where $\Sigma$ is defined by the equations

$$y = \sum_{k=1}^{n} \frac{\zeta_k}{x-a_k}, \quad t = \prod_{k=1}^{n} (x-a_k)^{N_k} . \tag{6.8}$$

The limits at infinity of these equations give

$$\begin{aligned} &x \longrightarrow \infty, \quad y \longrightarrow 0, && t \sim x^{N_1+\dots+N_n}, \\ &x \longrightarrow a_k, \quad y \longrightarrow \infty, && t \sim \zeta_k^{N_k} \prod_{i=1, i\neq k}^{n} (a_k - a_i)^{N_i} y^{-N_k} \end{aligned} \tag{6.9}$$

Matching the high energy $\mathcal{N}=2$ scales and low energy scales $\Lambda_k$ after integrating out the adjoint field we find agreement with the expected result $\zeta_k = \Lambda_k^3$ [5].

We now consider the effect of a geometric transition in the M theory picture, this corresponds to shrinking down the $\mathbf{P}^1$ cycle to zero. The $x^6$ direction in M theory becomes very small and we may consider $t$ as fixed on $\Sigma$. As in the case with quadratic superpotential, the curve $\Sigma$ makes a transition from a space curve into a plane curve. Eliminating $t$, we find there are $N_k$ supersymmetric vacua corresponding to the $N_k$ possible values of $\zeta_k$

and the curve is now described by

$$yW'(x) = \sum_{k=1}^{n} e^{2\pi i m_{k,i}/N_k} \prod_{l=1, l\neq k} (x - a_l) \; , \tag{6.10}$$

where $m_k = 1, \ldots, N_k$ for $k = 1, \ldots, n$. This is exactly the M theory lift of the generalised deformed conifold– see Section 5.3– with $\mu_k = \zeta_k \exp(2\pi i m_{k,i}/N_k) \prod_{l=1, l\neq n}(a_k - a_l)$.

# Chapter 7

# Metastable Vacua, Geometrical Engineering and MQCD Transitions

This chapter is based on the publication [28] done in collaboration with Radu Tatar. We further study the metastable vacua discussed in Sections 3.1 and 3.4 and the corresponding brane configurations discussed in Sections 4.5 and 5.4. The geometrically engineered configurations of [18] are translated into a IIA brane configuration by a T-duality. The MQCD transition is similar to that of Chapter 5 and [4–6]. A T-duality takes (anti)D5-branes into (anti)D4-branes on intervals between NS branes. The intervals between the branes and antibranes prevent their annihilation. There are two types of geometries to consider, one when all the $\mathbf{P}^1$ cycles are in the same homology class[1], and another when the $\mathbf{P}^1$ cycles are in different homology classes.

## 7.1 Metastable Vacua with Branes and Anti-Branes

We will use the following directions for the branes. The IIA brane configurations contain an NS brane in the directions (012345), an NS$'$ brane in

[1]This was the case studied in [18].

the (012389) directions and D4-branes in the (01237) directions. In type IIB the wrapped D5-branes are in the direction (012367) where $x^6$ is the angular direction of the $S^2$. In M-theory we use the following notations: $v = x_4 + ix_5$, $w = x_8 + ix_9$ and $t = \exp(-R^{-1}x_7 - ix_{10})$ where $R$ is the radius of the circle $S^1$ in the 11-th direction.

In [18] they discussed that not only can wrapped D5-branes be studied during geometric transitions, but also anti D5-branes. The usual geometric transition replaces wrapped D5-branes on two cycles $\mathbf{P}^1$ by fluxes on 3-cycles $S^3$. The wrapped D5-branes correspond to the UV limit of the field theory and the fluxes to the IR limit of the field theory as shown in Chapter 5. The mapping requires the identification of the number $N_k$ of wrapped D5-branes (the rank of the gauge group) with the flux of the $H_{RR}$ 3-form through the $S_k^3$ as

$$\int_{S_k^3} H_{RR} = N_k \ , \tag{7.1}$$

and the identification of the gluino condensate in the field theory with the size of the 3-cycle $S_k^3$

$$\int_{S_k^3} \Omega^{(3,0)} = S_k. \tag{7.2}$$

The new ingredient of [18] was to consider anti D5-branes wrapped on 2-cycles, this extends the equation (7.1) to negative $N$. The conjecture of [18] is that the geometric transition duality also holds for systems of D5-branes and anti D5-branes.

We can reformulate the new conjecture in terms of type IIA brane configuration by using the results of Chapter 5. The D5-branes wrapped on the $S^2$ cycle are mapped into D4-branes on the interval given by the radial direction of the $S^2$. The singular lines inside the resolved conifold are mapped into a pair of orthogonal NS branes.

If we instead wrap anti D5-branes, they are mapped into anti-D4 branes lying between two orthogonal NS branes. If we have both wrapped D5-branes and wrapped anti D5-branes, the system will be mapped into D4-branes and anti-D4 branes.

Let us consider a resolved geometry with multiple $S^2$ cycles and wrap $N_k$

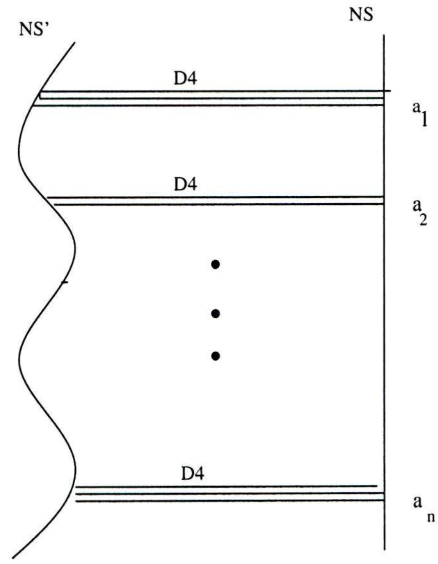


Figure 7.1: The T-dual configuration of D5 branes distributed among $n$ $\mathbf{P}^1$.

D5-branes on the $k$-th cycle and $N_{k'}$ anti D5-branes on the $k'$-th cycle. We now distinguish between two cases:

1. The $\mathbf{P}^1$ cycles are in the same homology class[2]. In this case the geometry is obtained by starting with a resolved $\mathcal{N}=2$, $A_1$ singularity and then deforming to an $\mathcal{N}=1$ theory by adding

$$W=\sum_{k=1}^{n+1}\frac{g_k}{k}\mathrm{Tr}\Phi^k\ , \tag{7.3}$$

where $\Phi$ is the unrestricted direction in the normal bundle. This gives a collection of $n$ resolved conifold $\mathcal{N}=1$ singularities which contain $n$ $P^1$ cycles in the same homology class.

We can also add D5 branes on each of the $\mathbf{P}^1$ cycles. After a T-duality this will become a straight NS brane and a curved NS$'$ brane [5] see figure 7.1.

In the limit $g_N\to\infty$ we have the situation of figure 7.2 where the curved NS$'$ changes into $n$ straight NS$'$ branes orthogonal to the NS

[2]This is the case considered in [18].

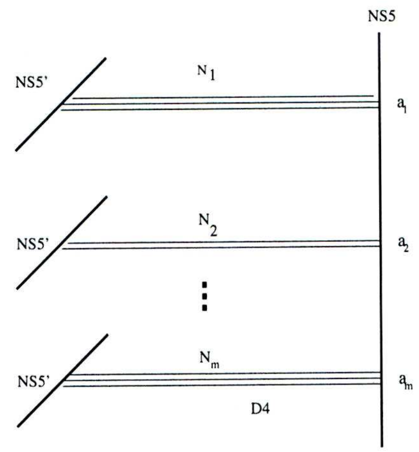


Figure 7.2: The brane configuration in the limit $g_n \to \infty$.

brane.

To reach the configuration of [18] we replace some stacks of D5-branes with stacks of anti D5-branes. Starting with $N_k$ D5-branes on the $k$-th cycle and $N_{k'}$ anti D5-branes on the $k'$-th cycle and performing a T-duality we get $N_k$ D4-branes at the position $a_k$ and $N_{k'}$ anti D4-branes at the position $a_{k'}$.

2. The $\mathbf{P}^1$ cycles are not in the same homology class. In this case we start with the resolution of an $\mathcal{N} = 2$, $A_n$ singularity and we wrap D5 branes on each of the $n$ $\mathbf{P}^1$ cycles. The T-dual is a configuration with D4-branes between $n$ intervals of parallel NS branes.

   By adding masses for the adjoint fields, we get an $\mathcal{N} = 1$ configuration with D4-branes between rotated NS branes. We can replace some of the D4-branes with anti D4-branes. In order to reduce the discussion to the one of the previous case consider that we have $N_k$ D4-branes between the $k$-th NS brane and the $k+1$-th NS brane and $N_{k'}$ anti D4-branes between the $k'$ NS brane and the $k'+1$ NS brane.

   The result of [65] for $k+1 = k'$ is that the two D4-branes and anti D4-

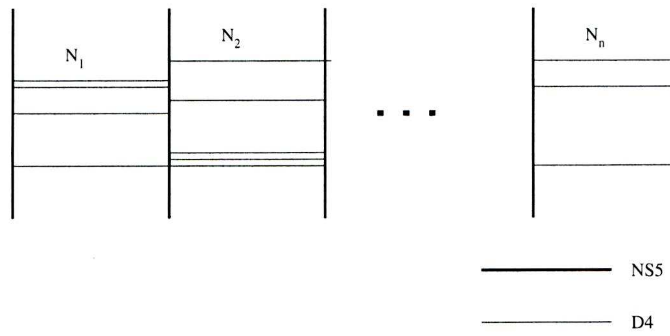


Figure 7.3: $A_n$ brane configuration: D5-branes wrapping $\mathbf{P}^1$ cycles are T-dualised to D4 branes between NS branes.

branes repel each other when adjacent. Nevertheless, if the D4-branes and anti D4-branes are not adjacent, the brane configuration then is stable and the stability also holds in the T-dual picture with wrapped D5-branes and anti D5-branes.

The IIA configuration can then be lifted to M-theory and we can then go through the MQCD transition of [4]. As the configuration of D4-branes and NS branes is lifted to a single M5 brane, the same holds for the configuration of anti D4-branes and NS branes.

We now perform the M-theory lift starting with the second case above which is simpler to describe. Remember that the D4-NS system is lifted to an M5 brane described by

$$v^{N_k} = t, \quad w^{N_k} = \xi^{N_k} t^{-1}, \quad vw = \xi_1 \ , \tag{7.4}$$

where $\xi_1$ is related to $\Lambda$, the dynamical scale.

The $N_k$ D4-branes are oriented as starting from the NS brane extended in the $v$-direction (therefore we have $v^{N_k} = t$) and ending on the NS$'$ brane extended in the $w$ direction (therefore we have $w^{N_k} = \xi_1^{N_k} t^{-1}$). For the case of $N_{k'}$ anti D4-branes between an NS brane and an NS$'$ brane the brane orientation reverses. The anti D4-brane starts on the NS$'$ brane and ends on the NS brane. The corresponding M5-brane is

$$w^{N_{k'}} = t, \quad v^{N_{k'}} = \xi_1^{N_{k'}} t^{-1}, \quad vw = \xi_2. \tag{7.5}$$

If we close the $S^2$-cycles that the D5-branes are wrapped on, in the T-dual picture this closes the intervals between the $k$-th NS brane and the $k+1$-th NS brane and between the $k'$-th NS brane and $k'+1$-th NS brane. The result is that the M5 brane (7.4) becomes a collection of $N_k$ planar M5 branes [4] described by

$$\Sigma_l : \quad t = t_0, \quad v\, w = \xi_1 \exp(2\pi i l/N_k), \; l = 0, \cdots, N_k - 1\;. \tag{7.6}$$

In the case of closing the cycles where anti D5-branes are wrapped, the only difference is replacing $\xi_1$ and $N_k$ with $\xi_2$ and $|N_{k'}|$ in (7.6):

$$\Sigma_n : \quad t = t_1, \quad v\, w = \xi_2 \exp(2\pi i l/|N_{k'}|), \; l = 0, \cdots, |N_{k'}| - 1\;, \tag{7.7}$$

where $|N_{k'}|$ is the absolute value of the flux due to the $k'$ anti D4-branes.

The only question is how are $\xi_1$ and $\xi_2$ related. For the D4-branes the value of $\xi_1$ is

$$\xi_1 = \Lambda_0^3 \exp(-\frac{2\pi\ i\ \alpha}{N_k})\;, \tag{7.8}$$

where $\Lambda_0$ is the cut-off scale and $\alpha$ is the bare coupling constant is defined by

$$\alpha = -\frac{\theta}{2\pi} - i\frac{4\pi}{g_{YM}^2}. \tag{7.9}$$

The Yang-Mills coupling constant $g_{YM}$ appearing in (7.9) may be written in terms of the geometry and string constants as

$$\frac{1}{g_{YM}^2} = \frac{\Delta L}{g_s l_s}\;, \tag{7.10}$$

where the value of $\Delta L$ measures the distance between the two NS branes. The difference in using D4-branes and anti D4-branes is that in the anti-brane case the measurement of $\Delta L$ is in opposite direction compared to the case of D4-branes. Thus[3] if for a D4-brane $\alpha$ is given by (7.9), the corresponding coupling constant for anti D4-branes is given by $\bar{\alpha}$. This implies that for anti D4-branes the value of $\xi_2$ is given by

$$\xi_2 = \Lambda_0^3 \exp\,(-\frac{2\pi\ i\ \bar{\alpha}}{|N_{k'}|}). \tag{7.11}$$

The main result of [4] was that after the MQCD transition, the value of $\xi_1$ is related to the size of the $S^3$ in the deformed geometry. Equation (7.6) reduces exactly to the deformed conifold when reducing from M-theory to type IIA theory, with the size of the $S^3$ being $\xi_1\exp(2\pi il/N_k)$. Because of the relation between $\xi_1$ and size of the $S^3_k$, we see that the geometric transition conjecture (see Chapter 5) holds if the gluino condensates (identified with the size of the $S^3_k$) for the gauge theories are

$$< S_k >= \Lambda_0^3 \exp(-\frac{2\pi i\alpha}{N_k}) \exp(2\pi il/N_k),\ l = 0, \cdots, N_k - 1. \tag{7.12}$$

The same thing holds for the case of anti D4-branes. The curve (7.7) reduces to a deformed conifold with the size of the $S^3_{k'}$ being $\xi_2\exp(2\pi il/|N_{k'}|)$. There is similarity between the deformation of the geometry with cycles with positive and negative fluxes. The relation between $\xi_2$ and $S_{k'}$ implies that the gaugino condensate in this case is

$$< S_{k'} >= \Lambda_0^3 \exp(-\frac{2\pi i\bar{\alpha}}{N_{k'}}) \exp(2\pi il/|N_{k'}|),\ l = 0, \cdots, N_{k'} - 1. \tag{7.13}$$

We now consider the first case when the $\mathbf{P}^1$ cycles are in the same homology class. In the $g_N \to \infty$ limit, the D4-branes and anti D4-branes end on the NS brane at the positions $a_k$ and $a_{k'}$. For the case of two stacks of D4 branes, the M5 brane would have the form:

$$t = (v - a_k)^{N_k}(v - a_{k'})^{N_{k'}}, w = \frac{\xi_1}{v - a_k} + \frac{\xi_2}{v - a_{k'}} \tag{7.14}$$

where $\xi_i$ are equal to $\Lambda_i^3$ with $\Lambda_i$ being the dynamical scales of the $\mathcal{N} = 1$

[3]Measuring $\Delta L$ from left to right.

theories. The $\Lambda_i$ are related to the $\mathcal{N}=2$ scales by the threshold condition

$$\Lambda_k^3 = g_{n+1}\Lambda_{\mathcal{N}=2}^{2N/N_k}(a_k - a_{k'})^{1-2N_{k'}/N_k};\ \Lambda_{k'}^3 = g_{n+1}\Lambda_{\mathcal{N}=2}^{2N/N_{k'}}(a_{k'} - a_k)^{1-2N_k/N_{k'}} \tag{7.15}$$

This formula is obtained after integrating out the adjoint fields which have mass $g_{n+1}(a_k - a_{k'})$ and the W-bosons which have mass $(a_{k'} - a_k)^{-2N_k/N_{k'}}$. The mass of the massive adjoint field is unchanged by replacing the D5-branes with anti D5-branes but the W-boson masses change. The change is due to the change of orientation of branes when changing from D-branes to anti D-branes. There is also a change $\alpha_2 \rightarrow \bar{\alpha_2}$ as discussed above.

[18] shows an explicit identification between field theory and geometrical quantities. As discussed in [5], the problems which arise in making the same identifications in the MQCD transitions are due to the fact that the geometrical curve is hyperelliptic and the MQCD curve is rational. The only case when the same identification can be made in MQCD is in the case of quadratic superpotentials for the adjoint field, which reduces to the second case described above.

In the next section we turn to the case of metastable vacua from rotated branes.

## 7.2 Metastable Vacua with Branes at Angles

In this section we consider the metastable vacua discussed in Section 3.1. The brane configuration and the MQCD picture were studied in [19–21] as well as Section 4.5. The work of [21] arrived at a negative conclusion concerning the possibility of having an MQCD picture for such metastable vacua. We will argue that a more general framework of deformations of $A_n$ singularities might be needed in order to obtain such an MQCD picture. This more general framework will be the subject of the next chapter.

In terms of brane configurations, the electric description has the same NS, NS′ and electric D4-branes as in the previous section but also some semi-infinite D4 branes ending on either the NS brane or the NS′ brane. For the D4-branes ending on the NS brane, the distance between the $N_c$ gauge

D4-branes and the semi-infinite flavour D4-branes is identified with the mass of the flavours[4]. For the D4-branes ending on the NS′ brane, the distance between the $N_c$ gauge D4-branes and the semi-infinite flavour D4-branes is identified with the vacuum expectation value of the meson $M$ .

The brane configuration is shown in Figure 7.4. The angle $\theta$ is the angle between the NS′ brane and the $w$ direction. tan$\theta$ is identified with the mass of the $\mathcal{N} = 2$ adjoint field. In what follows we will take $\theta = \pi/2$, *i.e.* the adjoint field has infinite mass.

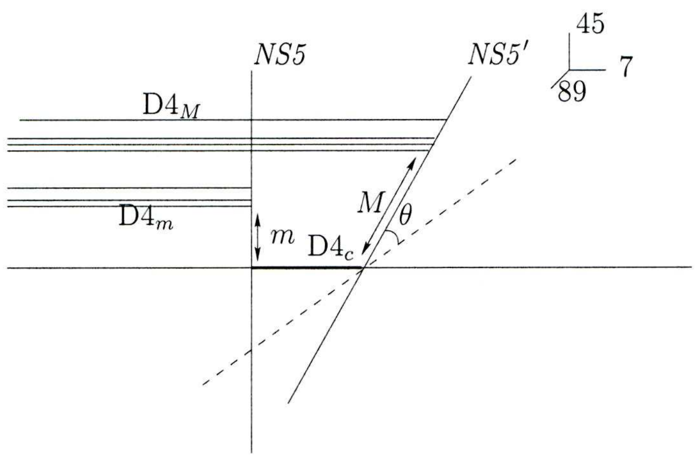


Figure 7.4: Brane construction .

After a Seiberg duality, the position of the NS and NS′ branes interchange. In considering the moduli space of vacua, there is a big difference between having massive quarks with a mass bigger or a mass lower than the scale. If the masses are larger than the scale, the quarks are integrated out and we get the scale

$$\tilde{\Lambda}_e^3 = \Lambda_e^{3N_c - N_f/N_c} (\prod \mu)^{1/N_c} , \tag{7.16}$$

[4]This is due to the fact that the $\mathcal{N} = 1$ brane configuration comes from an $\mathcal{N} = 2$ configuration by rotating the NS′ brane. The direction of the NS brane describes the Coulomb branch of the $\mathcal{N} = 2$ theory (the vacuum expectation value of the scalar field). Thus, moving on the Coulomb branch gives a vacuum expectation value to the adjoint field $\Phi$ and a mass to the fundamental quarks following from the coupling $Q\Phi\tilde{Q}$.

in the electric description and scale

$$\tilde{\Lambda}_m^3 = \Lambda_m^{3\tilde{N}_c - N_f/\tilde{N}_c} (\prod \mu)^{1/\tilde{N}_c} , \tag{7.17}$$

in the magnetic description where $\tilde{N}_c = N_f - N_c$. As considered in [5], the lift of this brane configuration to M-theory is an M5 brane dependent on $\mu$.

In the case of the non-supersymmetric vacua of [15], the position of the NS and NS′ branes interchange but the exchange between the masses of the electric quarks and the masses of the magnetic quarks does not hold. This is due to the fact that the magnetic quarks are massless but instead have vacuum expectation values. The mass of the electric quarks is mapped into the vacuum expectation values of the magnetic quarks[5].

We now perform a Seiberg duality. In the IIA brane configuration language, the case of massless electric quarks was shown in [3,10,11] and Section 4.3. When the stack of $N_f$ electric flavour branes touches the stack of $N_c$ electric colour branes, we can bind together the $N_c$ colour branes with $N_c$ of the flavour branes and move them together with the NS′ brane in the $x^6$ direction, then in the $x^7$ direction, and finally back in the $x^6$ direction. What we get is the dual magnetic theory with $N_f - N_c$ colour D4-branes and $N_f$ flavour D4-branes.

There is no such explicit construction for the case of light flavours since there is no way to bind the $N_c$ colour D4-branes to the $N_f$ light flavour D4-branes without breaking supersymmetry.

The displacement of the NS brane direction is the same in both electric and magnetic pictures. In the magnetic picture, the flavour D4-branes are at an angle with respect to the colour D4-branes. The angle is given by

$$\tan(\alpha) = \frac{\mu}{\Delta L} , \tag{7.18}$$

where $\Delta L$ is the length of the colour D4 branes and is related to the field

[5]Since the mass of the magnetic quarks is measured by distances on the NS′ brane, the vevs of the magnetic quarks are measured on the NS brane.

theory coupling constant $g$ by

$$\frac{1}{g^2} = \frac{\Delta L}{g_s l_s} . \tag{7.19}$$

There are two choices to construct the brane configurations in IIB; either with geometry and NS flux or just with geometry.

### Geometry and NS flux

We start with IIB branes and NS flux. Consider the following setup in type IIB: wrap finite D5-branes on an $S^2$ with coordinates $(y,\ \theta_2)$ and wrap infinite D5-branes on a non-compact cycle with the same radial and angular coordinates. Add some $B_{NS}$ field in the $(y, \theta_1)$ directions on both the finite $S^2$ and the non-compact 2-cycle, where $\theta_1$ is a direction orthogonal to the $S^2$ cycle.

A T-duality in the $y$ direction takes the wrapped D5-branes into parallel D4-branes. It is well-known that a T-duality on the $y$ direction of the $T^2(y, \theta_1)$ in the presence of a $B_{NS}$ field determines a rotation of the $T^2$ by an angle

$$\tan\beta = B_{NS}(y, \theta_1) \tag{7.20}$$

This results in the coordinates $(y, \theta_1)$ being rotated by an angle $\beta$ into the coordinates $(y', \theta_1')$.

We denote the coordinates of the NS branes and the D4-branes by $(r, z, y', \theta_1', x, \theta_2)$, the NS branes being extended in the directions $(x, \theta_1)$ and the NS$'$ branes being extended in the $(z, r)$ directions. We rotate the direction $\theta'$ until it coincides with $\theta_1$. The semi-infinite D4-branes do not feel the effects of the rotation as they are extended in the $\theta_2$ direction. Hence, the introduction of the NS field does not give the required IIA configuration with rotated D4-branes.

### Geometry

Consider the resolved conifold. The small resolution of the conifold is covered by two copies of $\mathbb{C}^3$ with coordinates $(Z, X, Y)$ and $(Z', X', Y')$. The resolved conifold geometry is defined by

$$Z' = 1/Z, \quad X' = XZ, \quad Y' = YZ \;, \tag{7.21}$$

which has a compact 2-cycle $Z' = 1/Z$. If we wrap $N$ D5-branes on this compact 2-cycle we get an $SU(N)$ gauge theory.

We can also define non-compact holomorphic cycles. To do so, start with an $\mathcal{N} = 1$ deformed $A_3$ singularity which after resolution gives a collection of three resolved conifold geometries:

$$Z_i' = 1/Z_i, \quad X_i' = X_i Z_i, \quad Y_i' = Y_i Z_i, \quad i = 1, 2, 3 \;; \tag{7.22}$$

where $X_1 = X_2'$, $X_2 = X_3'$, $Y_1 = Y_2'$, $Y_2 = Y_3'$.

There are three compact 2-cycles given by $Z_i' = 1/Z_i$, $i = 1, 2, 3$. Keep the lines $X_1 = X_2'$ and $X_2 = X_3'$ unchanged while taking the lines $X_1', Y_1'$ and $X_3, Y_3$ to infinity. This keeps the second 2-cycle compact but changes the first and third compact 2-cycles into non-compact holomorphic cycles. In the following we make the change of variables:

$$X_1 = X_2' = X', Y_1 = Y_2' = Y' \quad \text{and} \quad X_2 = X_3' = X, \; Y_1 = Y_2' = Y. \tag{7.23}$$

The non-compact 2-cycles considered in this chapter are

$$Y = 0, \quad X = \text{mass} \quad \text{or} \quad Y' = 0, \quad X' = \text{vev}. \tag{7.24}$$

As discussed in [5], the Seiberg duality can be obtained by a birational flop in the geometric engineering. For the resolution of the conifold, this means an exchange of $X$ (respectively $Y$) and $Y'$ (respectively $X'$) in the resolution, together with an interchange of the left and right non-compact 2-cycles. The flop appears quite natural for massless flavours. However, for massive flavours the flop again exchanges the $X$ (respectively $Y$) and $Y'$ (respectively $X'$) directions but it is less clear on how to handle the non-compact cycles describing the flavours.

The metastable solution of [15] corresponds to light massive flavours in the electric theory and massless flavours with vacuum expectation values in the magnetic theory. We still want to view the Seiberg duality as a flop in a geometry where the colour D5-branes and the flavour D5-branes touch each other. To do this, we need to perform changes in the geometry such that it resembles the one considered in [3, 10, 11]. Consider a non-holomorphic deformation of the $\mathbf{P^1}$ cycle, together with a rotation of the line bundle described by the following. First move the North Pole of the $\mathbf{P^1}$ cycle along the direction $X'$ by a very small distance $\mu$. The projection from the North Pole, used to define the coordinate $Z'$, changes and this makes the transition function from the upper to lower coverings non-holomorphic. In terms of brane configurations, this means a rotation of the D4-branes by an angle (7.18). The axis $X'$ is rotated by the same angle (7.18), until it becomes tangent to the North pole of the $\mathbf{P^1}$ cycle. Note that since the value $\mu$ in (7.18) is very small, the angle $\alpha$ is also very small. The non-compact 2-cycle ending on $X'$ is also forced to rotate by the angle (7.18) and in the final configuration there is an alignment between the two stacks of D5-branes. The map between the initial and final holomorphic transition functions is given by

$$Z' = 1/Z \quad \rightarrow \quad \tilde{Z}' = 1/\tilde{Z}. \tag{7.25}$$

In the geometry $\mathbb{C}^3(\tilde{X}, \tilde{Y} = Y, \tilde{Z})$, the North pole of the $\mathbf{P^1}$ coincides with the South Pole of the infinite holomorphic 2-cycle.

The normal bundle is also altered by the non-holomorphic deformation of the $\mathbf{P^1}$ cycle in order to describe the proper embedding of the "new" $\mathbf{P^1}$ into the resolved conifold. The overall change is

$$\mathbb{C}^3(X', Y', Z') \quad \rightarrow \quad \mathbb{C}^3(\tilde{X}, \tilde{Y} = Y, \tilde{Z}). \tag{7.26}$$

The line bundle $X'$ rotates into the line bundle $\tilde{X}$, whereas the line bundle $\tilde{Y}$ remains unchanged. The size of the $\mathbf{P^1}$ cycle remains the same. This means that the gauge coupling constant does not change after the geometrical manipulations as we would expect.

We can now try to see the change (7.26) in the MQCD coordinates. As shown in [4], there is a one to one map between the geometrical coordinates and the MQCD coordinates when the brane configuration is lifted to eleven dimensions. This is given by

$$Z \leftrightarrow t, \quad X' \leftrightarrow v, \quad Y \leftrightarrow w \, . \tag{7.27}$$

Start with the usual M5-brane wrapped on a holomorphic curve, in the presence of massive matter:

$$t \;=\; w^{N_c-N_f}\left(w \;-\; \left(\frac{\Lambda_{\mathcal{N}=1}^{3N_c-N_f}}{\mu^{N_c-N_f}}\right)^{1/N_c}\right)^{N_f} \quad , \quad vw \;=\; (\Lambda_{\mathcal{N}=1}^{3N_c-N_f}\mu^{N_f})^{1/N_c} \, . \tag{7.28}$$

This can be rewritten in terms of only $t$ and $v$ as

$$v^{N_c}\, t = \Lambda_{\mathcal{N}=1}^{3N_c-N_f}(\mu - v)^{N_f}. \tag{7.29}$$

Now we perform the change in the complex structure caused by very small rotations of the angle (7.18) in the $(x^4, x^7)$ plane. When rotating the $\mathbf{P}^1$ cycle, the radial direction will not change in the limit of small angles *i.e.* small quark masses. If the origin of the $\tilde{X}', \tilde{Z}$ is chosen to be at the new point of intersection, a rotation of the line bundle takes us to

$$\tilde{v}^{N_c}\tilde{t} = \Lambda_{\mathcal{N}=1}^{3N_c-N_f}\tilde{v}^{N_f} \, , \tag{7.30}$$

in the limit of very small $\mu$ and $\tilde{v} \to \infty$. This is just the usual asymptotic NS region

$$\tilde{v} \to \infty, \quad w \to 0, \quad \tilde{t} \to \Lambda^{3N_c-N_f}\tilde{v}^{N_f-N_c}. \tag{7.31}$$

In the case of very small masses, the asymptotic regions $v \to \infty$ and $\tilde{v} \to \infty$ are identical so the small deformation of the complex structure is invisible.

The coordinate $w$ is unchanged by the above manipulations. The relation between the coordinate $\tilde{t}$ and $w$ is similar to the one in (7.28) and this tells us that the usual asymptotic NS$'$ region is obtained by

$$w \to \infty, \quad \tilde{v} \to 0, \quad \tilde{t} \to w^{N_c}. \tag{7.32}$$

The curve obtained is then similar to one with massless matter and we can now discuss the Seiberg duality in the modified geometry. We now have the situation of [3, 10, 11] and the Seiberg duality then proceeds as the usual flop where

$$\mathbb{C}^3(\tilde{X}, \tilde{Y} = Y, \tilde{Z}) \quad \leftrightarrow \quad \mathbb{C}^3(\tilde{Y}', \tilde{X}', \tilde{Z}'). \tag{7.33}$$

Now, if we want to see the effect of the flop in the original complex structure, we need to rotate back from the tilded coordinates to the original coordinates. This will leave the $\tilde{Y}'$ axis invariant but will change the $\tilde{X}'$ and $\tilde{Z}'$ axes. The North Pole of the $\mathbf{P^1}$ cycle is moved to its original position and its transition function is again $Z' = 1/Z$.

The rotation of the line bundle does not affect the non-compact 2-cycle which remains in the rotated complex structure *i.e.* it is non-holomorphic in the original coordinates. After the rotation, there is an angle between the D5-branes wrapped on the compact 2-cycle and the D5-branes wrapped on the non-compact 2-cycles. The tilting of the branes causes a tachyon to appear between the two stacks of branes. There are two ways to cancel the tachyon. The first is to bind and rotate together the stacks of D5-branes, the alternative is to distance them such that the open string between them has no tachyonic mode.

The tachyonic mass $m_{tach}$ is related to the angle of rotation of the compact cycle with respect to the non-compact cycle by

$$m^2_{\text{tach}} = -\frac{\tan(\psi)}{l_s^2}\,, \tag{7.34}$$

where $\tan(\psi) = \mu/L_n$ and $L_n$ is the distance to the cut-off beyond which the normal deformations of the cycle inside the Calabi-Yau are frozen [26].

The final configuration is then obtained by combining the colour D5-branes with some of the D5-branes wrapped on the non-compact holomorphic 2-cycle ending on the $Y$ line of singularity. This gives D5-branes wrapped on a non-compact holomorphic 2-cycle ending on the $X'$ line of singularity. The other cycle is the non-compact non-holomorphic 2-cycle ending on the $Y$ line of singularity.

After the duality and recombination of branes, the configuration with $M = 0$, $q = \tilde{q} = 0$ is actually never obtained in the magnetic theory. This is due to the fact that the branes recombine before rotation back to the original geometry. In the magnetic theory we can turn on vevs for the field $M$ by displacing the non-compact cycle on the corresponding line bundle. If the non-compact cycle is infinitesimally displaced on the line bundle, there is an attractive force which determines a bound between the branes. If the displacement is made larger, the stacks of D5 branes tend to reject each other and the theory goes to the supersymmetric vacua.

# Chapter 8

# Metastable Vacua and Complex Deformations

This chapter is based on the publication [29] done in collaboration with Radu Tatar. In this chapter we present a general recipe to be used for dealing with the models with D5 branes wrapped on $P^1$ cycles of deformed ADE singularities. The T-dual of these types of configurations are D4 on intervals between NS branes. Before going any further, we make a clear distinction between two types of geometries:

1. **Deformations of the resolved $A_1$ singularity.**

   By wrapping $N$ D5 branes on the $P^1$ cycle of the resolved $A_1$ singularity, we get an $\mathcal{N} = 2$, $SU(N)$ gauge theory on the D5 branes. The field theory contains an adjoint field $\Phi$, by adding a superpotential $W(\Phi)$ of degree $m + 1$ in $\Phi$ such that

   $$W'(\Phi) = g_{m+1} \prod_{i=1}^{m} (\Phi - a_i) \, , \tag{8.1}$$

   we get an $\mathcal{N} = 1$, $\prod_{i=1}^{m} SU(N_i)$ gauge theory with the D5 branes now wrapped on $m$ $P^1$ cycles located at $\Phi = a_i$. We note that in this case the $m$ $P^1$ cycles are in the same homology class.

   We can also wrap anti D5 branes and get a non-supersymmetric system as done in [18]. The D5 branes and anti D5 branes can annihilate each other by overcoming the barrier determined by the separation of the $P^1$ cycles in the same homology class[1].

2. **Deformations of resolved $A_n$ singularities.**

   By wrapping $N_i$ D5 branes on the $n$ $P^1$ cycles of the resolved $A_n$ singularity, we get an $\mathcal{N} = 2$, $\prod_{i=1}^n SU(N_i)$ theory with fields transforming in the bifundamental representation between neighbouring groups. We consider adding a quadratic superpotential for all the adjoint fields of the groups $\prod_{i=1}^n SU(N_i)$. We must take into account several considerations concerning deformations of the geometry. To smoothen out the resulting geometry we need $n^2$ deformations [6]. $n(n+1)/2$ of the deformations are $P^1$ cycles which can be taken through a geometric transitions and these correspond to normalisable deformations of the geometry. $n(n-1)/2$ of the deformations do not correspond to $P^1$ cycles but to $S^3$ cycles which are non-normalisable in the geometry after a geometric transition. The non-normalisable deformations measure the distance between the various $P^1$ cycles.

   For a manifold with a certain intersection number for the $P^1$ cycles, a change in the intersection number is related to a change of complex structure. This change in complex structure is exactly realised by turning on the non-normalisable complex deformations.

The distances between the various $P^1$ cycles correspond to the masses or vacuum expectation values for the bifundamental fields. The masses or vevs are functions of the coefficients in the tree level superpotential deforming the $\mathcal{N} = 2$ theory so the coefficients of the superpotential fix the sizes of the non-normalisable deformations.

The non-normalisable deformations are displacements of the flavour cycles with respect to colour cycles on the common direction of their normal bundles. For $N_f > N_c$, we go to the Seiberg dual where the configurations contain branes and antibranes. The non-normalisable deformations are forced to change by brane-anti brane annihilation which determines a combination of some of them with the normalisable deformations.

[1]The T-dual for this was discussed in [28, 66].

The main point is the following: the geometric Seiberg dual is a flop or a Toric duality for toric manifolds of the original geometry. This holds nicely in the case of massless flavours with zero vev. In this case, even though we have D5 branes and anti D5 branes in the magnetic theory, the two stacks of branes are on top of each other and the tachyon condensation ensuring a stable system does not change the geometry. If there is a change of the complex structure of the electric theory geometry caused by a change of intersection number then, after a Seiberg duality, we have a magnetic theory with displaced D5 and anti D5 branes. The following tachyon condensation determines the closure of some of the non-normalisable cycles. Since the D5 branes are wrapped on normalisable cycles, this can be obtained by a recombination of some of the normalisable and non-normalisable cycles. The geometry will now contain unchanged $P^1$ cycles which are holomorphic embedded in the geometry and cycles which have been combined with non-normalisable cycles. These are non-holomorphic in the original geometry and are metastable cycles.

We can also compare our results to the ones of [18]. In their case there is also a clear distinction between the normalisable deformations and non-normalisable deformations: the total number of deformations is $2m-1$, where $m$ is the highest power in $W'(\Phi)$. The number of normalisable deformations is $m$ and the number of non-normalisable deformations is $m-1$. The main difference between our case and the one of [18] is that in [18] they considered the case of $A_1$ singularity deformed by a general superpotential. They obtain many $P^1$ cycles which are in the same homology class and are separated by non-normalisable deformations. By wrapping D5 branes and anti D5 branes on the cycles, the branes and anti branes can cancel each other but the Hodge number $h_{11} = 1$ is not modified and there is no recombination of cycles. As discussed in [67], the brane configuration involves a straight NS brane and a curved NS brane along the direction $W'(x)$ where $W(x)$ is the tree level superpotential.

The intersection of branes and anti branes occurs at $W'(x) = 0$ and the D4 branes are located at these points in the direction $x$. The tachyon condensation occurs first on the straight NS brane and the stacks of D4 and anti D4 branes are forced to intersect on the straight NS brane. The bending of the D4 branes then propagates to the curved NS brane and the ends of the (anti) D4 branes on the curved NS brane can touch each other by an increase in their energy and by moving on the curved NS brane. When the ends on the curved NS brane coincide, there is total annihilation and the vacuum is a configuration without any D4 branes.

The situation is different for the deformation of the $A_n$ singularity. In this case the D5 branes and anti D5 branes are wrapped on cycles in different homology classes separated by non-normalisable deformations. Due to the tachyon between the D5 branes and anti D5 branes, there is a recombination of the cycles. In type IIA brane configurations, there are D4 branes and anti D4 branes with one end on the same NS brane and the other ends on different NS branes. The phenomenon proceeds as in [27], the ends of D4 and anti D4 branes on the same NS brane are connected after tachyon condensation. Since the other ends of the branes are on different NS branes, there is no total annihilation but the cycles recombine in order to minimise the action of the branes.

The normalisable cycles in some homology classes are unchanged and the normalisable cycles in other homology classes combine with the non-normalisable and become non-holomorphically embedded in the geometry.

## 8.1 Complex Structure Deformations

In this section we will spell out the identification between the geometrical quantities and the field theory quantities. We then see how the non-normalisable deformations are related to the metastable non-SUSY vacua and SUSY vacua.

Some particular cases of interest are:

- **$A_2$ quiver deformed by a quadratic superpotential**
  Here there are four complex deformations, out of which three are normalisable deformations corresponding to gluino condensates and the other is a non-normalisable deformation corresponding to parameters

of the tree level superpotential.

At the level of field theory, this case was discussed in [68] as an $\mathcal{N} = 2$, $SU(N_1) \times SU(N_2)$ supersymmetric theory broken to $\mathcal{N} = 1$ by adding masses $m_1$ and $m_2$ for the two adjoint fields. The $\mathcal{N} = 1$ theory has an $SU(\tilde{N}_1) \times SU(\tilde{N}_2) \times SU(\tilde{N}_3)$ gauge group. The bifundamentals of the initial $SU(N_1) \times SU(N_2)$ theory become $\mathcal{N} = 1$ fields which are either massive with mass $\mu$ or have a vacuum expectation value $\nu$ given by

$$\nu = \frac{\mu}{\frac{1}{m_1} + \frac{1}{m_2}} , \tag{8.2}$$

where $m_i$ are the mass of the adjoint fields labelled in an obvious manner. Let us consider the case $\tilde{N}_3 = 0$ and denote $\tilde{N}_1 = N_c$ and $\tilde{N}_2 = N_f$. This is the case with a gauge group $SU(N_c) \times SU(N_f)$ with massive bifundamentals. If we then take the coupling constant of $SU(N_f)$ to zero, this will provide us the case of an $\mathcal{N} = 1$ theory with $SU(N_c)$ gauge group and $N_f$ massive flavours with a mass $\mu$.

In terms of the geometrical deformations, the three normalisable complex deformations correspond to the three possible gluino condensates of the $SU(\tilde{N}_i), i = 1, 2, 3$ groups. The non-normalisable deformation is measured by the value of $\nu$. In this case the values of $m_i$ are very large and this implies a very small value for $\mu$. Therefore the condition of [15] (very small value for the mass of the flavours) is satisfied by the geometry.

- **$A_3$ quiver deformed by a quadratic superpotential**
  Here we have nine complex deformations, out of which six are normalisable deformations corresponding to gluino condensates and three are non-normalisable deformations corresponding to parameters in the tree level superpotential. The field theory obtained by resolving the $A_3$ singularity is an $SU(N_1) \times SU(N_2) \times SU(N_3)$ theory with two pairs of bifundamental fields. The three non-normalisable deformations can be either masses or expectation values for the bifundamental fields.

- **$A_1$ quiver deformed by a cubic superpotential**

Here we have three complex deformations out of which two are normalisable deformations corresponding to gluino condensates and one is a non-normalisable deformation corresponding to parameters in the tree level superpotential.

### 8.1.1 Seiberg Dualities and Geometry Deformations

Consider an $\mathcal{N}=1$ $SU(N_c)$ gauge theory with $N_f > N_c$ flavours $Q, \tilde{Q}$. The Seiberg dual is an $SU(N_f - N_c)$ gauge theory with $N_f$ dual flavours $q, \tilde{q}$, a gauge singlet in the adjoint representation of the flavour group $SU(N_f)$, $M$ and superpotential $W_{mag} = qM\tilde{q}$. The electric flavours can be either massive or have expectation values. The corresponding situation in the dual theory is the following:

- **Give an expectation value to $n$ of the flavours**
  This breaks the gauge group to $SU(N_c - n)$ and there are now $N_f - n$ fundamental flavours. The Seiberg dual theory is an $SU(N_f - N_c)$ gauge theory with $N_f - n$ dual flavours $q, \tilde{q}$ and a gauge singlet $M$ in the adjoint representation of the flavour group $SU(N_f - n)$. The gauge singlet loses components by getting an expectation value and the superpotential $W_{mag}$ implies that $n$ of the initial $N_f$ flavours in the dual theory acquire a mass.

- **Give a mass to $n$ of the flavours**
  If the mass is bigger that the dynamical scale, then the flavours can be integrated out and the theory becomes an $SU(N_c)$ gauge theory with $N_f - n$ fundamental flavours. If $N_f - n > N_c$, the Seiberg dual theory still exists but it is now an $SU(N_f - N_c - n)$ gauge theory with $N_f - n$ dual flavours $q, \tilde{q}$ and a gauge singlet in the adjoint representation of the flavour group $SU(N_f - n)$. The rest of $n$ flavours have acquired a vacuum expectation value which determines a linear term in the superpotential for some components of the singlet M. If we give mass to more than $N_f - N_c$ of the flavours then the dual gauge group is completely Higgsed. We can give mass to all the flavours and keep the mass under

the dynamical scale so that the flavours are light.

The geometrical interpretation of the Seiberg duality is as a flop in the geometry. The $SU(N_c)$ colour group is identified with $N_c$ D5 branes wrapping a finite $P^1$ cycle whereas the $SU(N_f)$ group is identified with $N_f$ D5 branes wrapping a holomorphic non-compact 2-cycle. The colour and flavour cycles have a common direction in their normal bundle. There are bifundamental fields of the group $SU(N_c) \times SU(N_f)$ which become the fundamental flavours if the $SU(N_f)$ factor is a flavour group. If the two cycles touch, the bifundamentals are massless and have zero expectation value. If there is a displacement between the cycles on the normal bundle directions, the bifundamentals have either mass or expectation value.

The masses and expectation values of the bifundamentals are related to the non-normalisable deformations of the geometry. The number of these deformations should be the same in both the electric and magnetic theories. The embedding of the colour cycle $P^1$ into its normal bundle implies the existence of two complex lines orthogonal to it. The two complex lines touch the $P^1$ cycle at its North Pole and South Pole.

When we view the Seiberg duality as a flop in the geometry the two complex lines described above are interchanged. If we choose to associate masses with the displacement on one such line and associate vacuum expectation values to displacement on the other line, then the Seiberg duality reverses this convention. Thus, we see that the map of masses into vacuum expectation values and *vice versa* appears very naturally in the geometry.

The relation between the non-normalisable deformations and the expectation values for the flavours (same as having a linear term for the singlet) or masses for the flavours in the Seiberg dual theories is useful for checking that we have all the terms in the superpotential.

Consider the $A_3$ singularity deformed by a quadratic superpotential. The $\mathcal{N} = 2$ gauge theory of the wrapped D5 branes becomes an $\prod_{i=1}^{3} SU(N_i)$ gauge theory with the bifundamentals $Q_1, \tilde{Q}_1$ in the $(N_1, \bar{N}_2), (\bar{N}_1, N_2)$ and $Q_2, \tilde{Q}_2$ in the $(N_2, \bar{N}_3), (\bar{N}_3, N_2)$ representations of the gauge groups. If the middle 2-cycle (identified with the $SU(N_2)$ part) is finite and the other cycle semi-infinite, then the theory can be viewed as an $\mathcal{N} = 2$, $SU(N_2)$ gauge theory with $N_1 + N_3$ fundamental flavours. There are three non-normalisable deformations which can be associated to either

- Mass or vacuum expectation values for the $Q_1, \tilde{Q}_1$ quarks. This requires the existence of the term $Q_1 M_{11} \tilde{Q}_1 + M_{11}$ in the superpotential.

- Mass or vacuum expectation values for the $Q_2, \tilde{Q}_2$ quarks. This requires the existence of the term $Q_2 M_{22} \tilde{Q}_2 + M_{22}$ in the superpotential.

- Vacuum expectation for the $Q_1 Q_2$. This requires the existence of the term $Q_1 M_{12} \tilde{Q}_2$ in the superpotential. The complete superpotential for all the deformations is

$$W = aQ_1 M_{11} \tilde{Q}_1 + bQ_2 M_{22} \tilde{Q}_2 + cQ_1 M_{12} \tilde{Q}_2 + dM_{11} + eM_{22} \ , \tag{8.3}$$

where the coefficients are functions of the electric variables and dynamical scales of the theories.

Now consider the case of an $A_5$ singularity and consider the Seiberg duality only for the even modes as in [69]. For an $A_5$ singularity deformed by a quadratic superpotential there are ten non-normalisable deformations. The $\mathcal{N} = 2$ gauge group is $\prod_{i=1}^{5} SU(N_i)$ with four pairs of bifundamental fields $Q_i, \tilde{Q}_i$, $i = 1, \cdots, 4$. There are ten possible vacuum expectation values for the bifundamental fields and their products $Q_i Q_{i+1}$, $Q_i Q_{i+1} Q_{i+2}$, $Q_1 Q_2 Q_3 Q_4$.

If we take the Seiberg dual of the second and fourth gauge groups, we find a collection of two $A_3$ singularities deformed by quadratic superpotentials. Therefore only six of the non-normalisable deformations are visible *i.e.* the vacuum expectation values or masses for $Q_1, Q_2, Q_3, Q_4$ and vacuum expectation values for $Q_1 Q_2, Q_3 Q_4$. The superpotential allowing these deformations is

$$\begin{aligned} W = \ & Q_1 M_{11} \tilde{Q}_1 + Q_2 M_{22} \tilde{Q}_2 + Q_3 M_{33} \tilde{Q}_3 + Q_4 M_{44} \tilde{Q}_4 + \\ & Q_1 M_{12} \tilde{Q}_2 + Q_3 M_{34} \tilde{Q}_4 + M_{11} + M_{33} + M_{55} \ . \end{aligned} \tag{8.4}$$

The main question of this work is how to handle these non-normalisable deformations for the models with metastable non-supersymmetric vacua. We turn on non-normalisable deformations and see what the effect is in the geometry. We start by deforming the electric theory and see which corresponding non-normalisable deformations are present in the magnetic theory side. We will see that the non-normalisable deformations combine with some of the normalisable deformations to form new cycles.

## 8.2 $\mathcal{N} = 1,\ SU(N_f) \times SU(N_c)$ Model

Consider the $\mathcal{N} = 2$, $SU(N_f) \times SU(N_c)$ model and deform it in the following way breaking $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$:

$$W_{ele} = \frac{1}{2}\tilde{\mu}\tilde{\Phi}^2 + \frac{1}{2}\mu\Phi^2 + \tilde{Q}(\Phi + \tilde{\Phi})Q + \xi\Phi + \tilde{\xi}\tilde{\Phi}\ . \tag{8.5}$$

The $F$-term equations are

$$F_Q = \tilde{Q}\Phi + \Phi\tilde{Q}\ , \tag{8.6}$$

$$F_{\tilde{Q}} = \Phi Q + Q\tilde{\Phi}\ , \tag{8.7}$$

$$F_{\Phi} = \mu\Phi + Q\tilde{Q} + \xi\ , \tag{8.8}$$

$$F_{\tilde{\Phi}} = \tilde{\mu}\tilde{\Phi} + \tilde{Q}Q + \tilde{\xi}. \tag{8.9}$$

As discussed in [70], there are several solutions to these equations:

1. The electric theory corresponds to $\mu\tilde{\mu} \neq 0$ when both adjoint fields are massive.

   If the linear terms are zero $\xi = \tilde{\xi} = 0$, the solution with $SU(N_f) \times SU(N_c)$ group (*i.e.* no expectation value for the fundamental flavours) is $\Phi = \tilde{\Phi} = 0$.

   If the linear terms are not zero $\xi\ \tilde{\xi} \neq 0$, the situation changes. $Q$ and $\tilde{Q}$ can be simultaneously diagonalised by a colour rotation and they assume the form:

$$Q = \mathrm{diag}(q_a), \qquad \tilde{Q} = \mathrm{diag}(\tilde{q}_a). \tag{8.10}$$

The equations of motion imply that $Q\tilde{Q}$ have at most one non-vanishing eigenvalue. This eigenvalue corresponds to a displacement of the NS branes in the brane configuration picture or displacement of the cycles in the geometrical picture. These displacements can correspond to either masses or vevs for the fundamental fields.

2. The magnetic theory corresponds to $\mu \neq 0$ and $\tilde{\mu} = 0$ when one of the adjoint fields is massive, the other adjoint field becomes the meson singlet and remains massless.

   If the linear terms are zero $\xi = \tilde{\xi} = 0$, the expectation values for $Q, \tilde{Q}$ are zero so $\tilde{\Phi}$ is free to take any value. The $\tilde{\Phi}$ is the singlet meson field in the dual theory.

   If the linear term $\tilde{\xi} \neq 0$, the supersymmetric solution requires $N_c \geq N_f$. In the magnetic theory, the number of colours is $N_f - N_c$ and the number of massive flavours is $N_f$ so SUSY is broken by the rank condition *i.e.* the $F$-term equation for the field $\tilde{\Phi}$ does not have a solution [15].

By continuously changing the parameters from the case 1 to case 2 we may go from the electric theory to the magnetic theory by a Seiberg duality. We now want to describe the geometrical interpretation of this duality.

## 8.3 The Corresponding Geometry

In order to discuss the geometrical interpretation of the Seiberg duality for the model with light flavours, we start with a simpler one, the $\mathcal{N} = 1$, $SU(N_f) \times SU(N_c)$ theory with massless flavours shown in Figure 8.1.

### 8.3.1 Massless Flavours

We are going to discuss the electric and magnetic brane configurations for this theory. The T-duality between the brane configurations and geometry implies that we need to add an extra $S^1$ circle to the D4 branes in order to get D5 branes on an $S^1 \times$ interval, *i.e.* D5 branes on $P^1$ cycles. After

T-duality, the NS branes are mapped into lines of singularity living in the the normal bundle of the $P^1$ cycles. Even though we will only draw brane configurations, we understand that there is an extra $S^1$ which is the extra dimension of the D5 branes [4–6, 67].

In order to discuss the geometrical interpretation of the Seiberg duality we start with a resolved $\mathcal{N}=2,\ A_3$ singularity. Each $P^1$ cycle has a normal bundle:

$$X' = X, \quad Y' = YZ^2, \quad Z' = 1/Z \ . \tag{8.11}$$

We then deform the $\mathcal{N} = 2,\ A_3$ singularity into a collection of resolved conifold singularities, each one looking like

$$X' = XZ, \quad Y' = YZ, \quad Z' = 1/Z \ . \tag{8.12}$$

The number of normalisable deformations is six and the number of the non-normalisable deformations is three. If we close all the non-normalisable cycles, there are three normalisable $P^1$ cycles remaining which touch each other. In Figure 8.1, we denote the three $P^1$ cycles of the $\mathcal{N}=1$ resolved geometry by $A$, $B$ and $C$ from left to right. We always understand that the cycles A and C have very large volume as they are considered flavour cycles.

The electric brane configuration involves $N_f$ D5 branes wrapped on the A cycle and $N_c$ D5 branes wrapped on the B cycle.

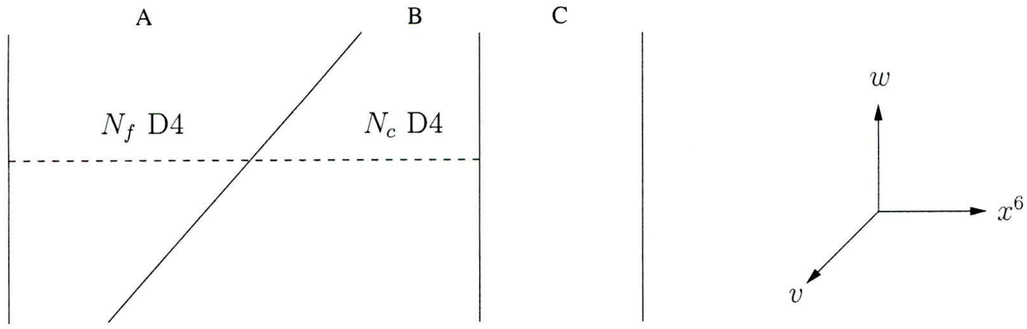


Figure 8.1: Electric configuration of branes with $N_f$ Massless Quarks

We now use the results of [12, 13, 62] which considered the Seiberg duality as a Toric duality. For the case in hand, the Toric duality implies shrinking down the $A$ cycle and blowing up the $C$ cycle. This is because the geometry can separated into a resolved conifold singularity and a resolution of a deformed $A_2$ singularity. This can be done in two ways and the results of [62] imply that the Seiberg duality is a Weyl reflection in the Dynkin diagram of the $A_3$. In terms of the cycles this means

$$A \to A' = A + B, \quad B' \to -B \ , \tag{8.13}$$

which satisfy the charge conservation condition

$$N_f A + N_c B = N_f A' + (N_f - N_c) B' \ . \tag{8.14}$$

The $A$ and $C$ cycles are interchanged by the flop since they touch the $B$ cycle at its North and South pole respectively, and the poles of the $B$ cycle are interchanged. The resulting picture is in Figure 8.2 where the order of cycles is now $A$, $B$ and $C$ from right to left.

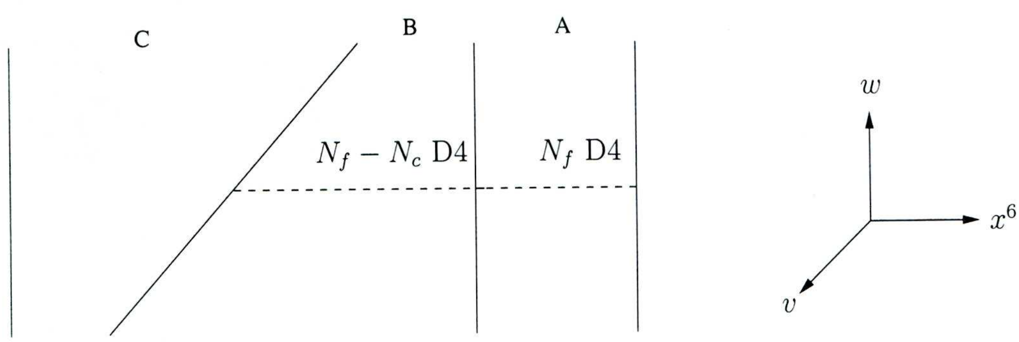


Figure 8.2: Magnetic configuration of branes with $N_f$ Massless Quarks

The $N_c$ D5 branes on the $B$ cycle are changed into $N_c$ anti D5 branes on the $B'$ cycle and the $N_f$ D5 branes on the $A$ cycle are changed into $N_f$ branes on the $A'$ cycle.

There is a tachyon condensation between the $N_c$ anti D5 branes on the $B'$ cycle and $N_c$ of the $N_f$ D5 branes on the $A'$ cycle. Since the D5 branes and the anti D5 branes are on top of each other, the cycles of the geometry remain unmodified by the tachyon condensation. The result is a configuration with $N_f - N_c$ D5 branes on the $B'$ cycle and $N_f$ D5 branes on the $A'$ cycle of the dual geometry.

The discussion for the case of massive quarks is more involved. The electric geometry is shown in Figure 8.3. To understand it we first need to make some observations.

### 8.3.2 Massive Flavours

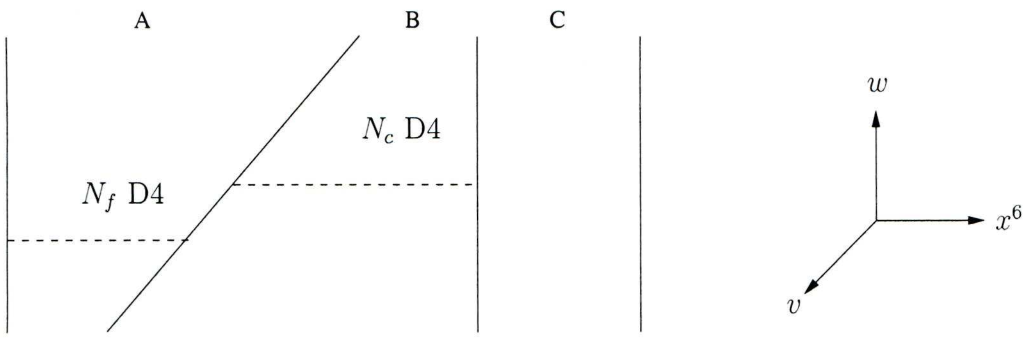


Figure 8.3: Electric configuration of branes with $N_f$ Massive Quarks

There are two ways to represent the fundamental matter of the $\mathcal{N} = 2$, $SU(N_f) \times SU(N_c)$ brane configuration.

1. One way involves three parallel NS branes with sets of $N_f$ D4 branes and $N_c$ D4 branes between them.

2. The second way is with two parallel NS branes together with $N_c$ D4 branes and $N_f$ orthogonal D6 branes.

To go to the $\mathcal{N} = 1$ theory we rotate the NS branes relative to each other, the rotation of one of the NS branes is related to giving a mass of $\tan\theta$ to the adjoint field, where $\theta$ is the rotation angle. For the second case, a rotation of the D6 branes is related to changing the Yukawa coupling by a factor $\cos\alpha$ where $\alpha$ is the rotation angle of the D6 branes relative to the NS branes.

In the configuration with only NS branes and D4 branes there are several ways to rotate the NS branes. A rotation of the middle or the rightmost NS brane by an angle $\theta$ implies a mass for the adjoint field equal to $\tan\theta$. A rotation of the leftmost NS brane by an angle $\alpha$ implies a factor $\cos\alpha$ in front of the Yukawa coupling. By keeping the middle NS brane unchanged and rotating the outer NS branes by the same angle $\theta$, this gives a mass $\tan\theta$ to the $\mathcal{N}=2$ adjoint field and also puts a factor $\cos\theta$ in the Yukawa coupling.

We can now understand the difference between Figures 8.1 and 8.3. We see that there is no way to directly go from one to the other since the stacks of D4 branes lie between non-parallel NS branes and are stuck. The only way to deform one into the other is to use the non-normalisable deformations of the theory. For an $A_2$ singularity there are three normalisable deformations and one non-normalisable deformation. The latter is related to adding a linear term in the $\mathcal{N}=2$ adjoint field to the superpotential which is related to turning on a mass or vacuum expectation value for the fundamental quarks.

As the Seiberg duality is a Toric duality, the magnetic picture should look like Figure 8.4.

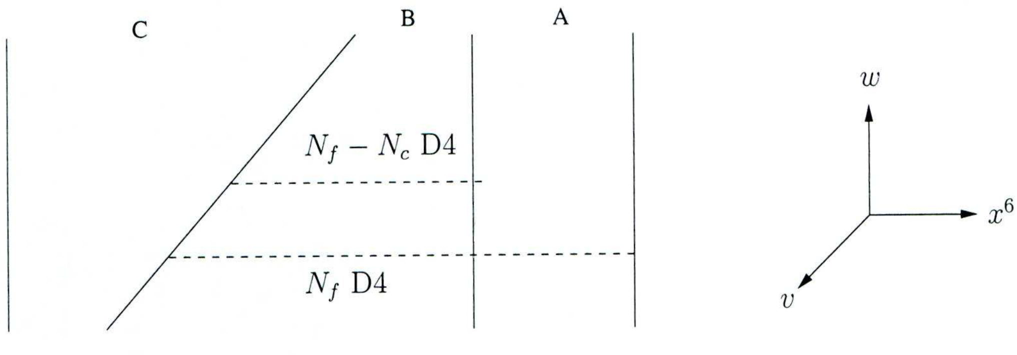


Figure 8.4: Magnetic configuration of branes from $N_f$ Massive Quarks

We now attempt to obtain the magnetic geometry from some non-normalisable deformation of an $\mathcal{N}=2$ theory. The deformation for the $\mathcal{N}=2$, $SU(N_f)\times SU(N_c)$ would be

$$W=\frac{m}{2}\Phi_1^2+\xi_1\Phi_1+\xi_2\Phi_2 \tag{8.15}$$

because there is no mass term for $\Phi_2$. Here we can identify $\Phi_2$ with the meson singlet of the Seiberg dual theory.

The effect of the first two terms in the superpotential is to create a displacement between the $A'$ and $B'$ cycles on the line of singularity corresponding to the right NS brane. Now let us put the corresponding $N_c$ anti D5 branes on the $B'$ cycle and $N_f$ D5 branes on the $A'$ cycle.

The effect of the tachyon condensation is more complicated than in the massless case because it has to undo the non-normalisable deformation. As argued in [27], the condensation occurs first on the left NS brane and then propagates to the right NS branes as a kink. The D4 branes bend in the directions of the NS branes [27]. In the geometric picture, this maps into a bending of the holomorphic $P^1$ cycle in its normal bundle directions.

The bending of the cycle containing $N_c$ wrapped D5 branes is T-dual to the bending of the $x^6$ direction of the D4 brane into the $x^5$ direction as discussed in [27]. A Dirac-Born-Infeld (DBI) equation of motion argument [27] gives the result of the bending as

$$\delta x = \frac{1}{2\,l}(y_1^2 \sin 2\theta_1 + y_2^2 \sin 2\theta_2) + l(\theta_1 + \theta_2)\ , \tag{8.16}$$

where $l$ is the characteristic scale, $x$ is the displacement of the $P^1$ cycle in the direction of the non-normalisable cycle, $y$ is the size of the two cycles and $\cos\theta_i = \frac{y_m}{y_i}$ where $y_m$ is the minimum of $y$. In the cases considered in [27], this solution was quite complicated and involved several types of transitions.

The simplification in the case in hand is due to the position of the rightmost NS brane being at infinity[2]. The mass of the fundamental flavours is also small so the $x$ displacement is small. In this case the dependence of $x$ on $y$ is monotonic [27] and there is no other maxima or minima. Therefore, the tachyon condensation after the flop for the case in hand will give two stacks of D5 branes, one stack of D5 branes $N_f - N_c$ wrapped on the $A'$ cycle and one stack of $N_c$ D5 branes wrapped on the deformation of the $A$ cycle.

We can also make a connection to M-theory. T-duality between wrapped D5 branes and IIA brane configurations followed by a lift to M theory allows a direct identification between the cycles of the IIB geometry and the cycles of

[2]The corresponding cycle is semi-infinite since it is a flavour cycle.

the Riemann surface of the M5 brane. The M5 brane has 1-cycles corresponding to the non-normalisable cycles in IIB. By closing the non-normalisable cycle, there are cycles in the Riemann surface which remain holomorphic and some which are not holomorphic. The non-normalisable deformation is the difference between the two types of cycles discussed in [20, 21].

## 8.4 $\mathcal{N}=1$, $SU(N_f - N_c) \times SU(N_c) \times SU(N_c)$ Model

In this section we consider the situation of [71]. Before doing so, we make some comments on the relation between configurations with flavours given by D6 branes and the configurations with flavours given by D5 branes wrapped on semi-infinite 2-cycles.

Consider first the configuration with NS branes and D6 branes. The $\mathcal{N}=2$ model includes an NS brane parallel to the D6 branes and orthogonal to the other NS branes. Let us now split the stack of D6 branes into one stack containing $n_1$ D6 branes and one stack containing $n_2$ D6 branes with $n_1 + n_2 = N_f$. We then rotate the right NS brane and the stack of $n_1$ D6 branes by an angle $\theta$. There is a repulsive force between D6 branes which are not parallel [3] so the rotation breaks the flavour group $SU(n_1 + n_2)$ into $SU(n_1) \times SU(n_2)$. If we denote the $n_1$ quarks by $Q_1, \tilde{Q}_1$ and the $n_2$ quarks by $Q_2, \tilde{Q}_2$ then the superpotential contains the terms

$$W \supset \cos\theta Q_1 \Phi \tilde{Q}_1 + \cos\theta Q_2 \Phi \tilde{Q}_2 + \frac{1}{2} \tan\theta \Phi^2 \ . \tag{8.17}$$

By integrating out the adjoint field we get the quadratic coupling

$$Q_1 \tilde{Q}_1 Q_2 \tilde{Q}_2 \ . \tag{8.18}$$

We then take the stack of $n_1$ D6 branes to the right infinity and the stack of $n_2$ D6 branes to the left infinity. There is a brane creation due to the Hanany-Witten effect [55], the $n_1$ D6 branes are now connected by $n_1$ D4 branes to the left NS brane and the $n_2$ D6 branes are connected by $n_2$ D4 branes to the right NS brane.

We can then exchange the model with D6 brane flavours for one with D4 branes between NS branes on semi-infinite intervals. To perform a T-duality of this configuration we replace the D4 branes with D5 branes wrapped on non-compact cycles and the NS branes with the normal bundle to the $P^1$ cycles. In the $\mathcal{N} = 2$ model, all the four NS branes are parallel. In the rotated model, the rotation of the $n_1$ D6 branes by an angle $\theta$ is equivalent to the rotation of the rightmost NS by the same angle $\theta$. The $\mathcal{N} = 1$ model discussed above has the first and the third NS branes parallel to each other and rotated by an angle $\theta$ with respect to the second and fourth NS branes which are parallel and unrotated.

The unrotated IIB geometry is the $\mathcal{N} = 2,\ A_3$ model. We can solve the singularity and wrap branes on the cycles to get an $\mathcal{N} = 2, SU(N_f - N_c) \times SU(N_c) \times SU(N_c)$ model. The two above rotations imply that all the adjoint fields will get a mass equal to $\mu = \tan\theta$ causing a breaking of supersymmetry to $\mathcal{N} = 1$. There is also a change in the coupling between the bifundamentals and adjoint fields , which acquires an extra factor $y = \cos\theta$. We can also add a linear term in the adjoint fields of the type $\xi_i \Phi_i$. After these deformations the superpotential becomes

$$W_{ele} = \sum_{i=1}^{3} \frac{1}{2}\mu\Phi_i^2 + y\tilde{Q}_1(\Phi_1 + \Phi_2)Q_1 + y\tilde{Q}_2(\Phi_2 + \Phi_3)Q_2 + \sum_{i=1}^{3} \xi_i\Phi_i \ . \quad (8.19)$$

The $F$-term equations are

$$F_{\tilde{Q}_1} = \Phi_1 Q_1 + Q_1\Phi_2, \quad F_{Q_1} = \tilde{Q}_1\Phi_1 + \Phi_2\tilde{Q}_1 \ , \quad (8.20)$$

$$F_{\tilde{Q}_2} = \Phi_2 Q_2 + Q_2\Phi_3, \quad F_{Q_2} = \tilde{Q}_2\Phi_2 + \Phi_3\tilde{Q}_2 \ , \quad (8.21)$$

$$F_{\Phi_1} = \mu\Phi_1 + yQ_1\tilde{Q}_1 + \xi_1, \quad (8.22)$$

$$F_{\Phi_2} = \mu\Phi_2 + y\tilde{Q}_1Q_1 + Q_2\tilde{Q}_2 + \xi_2, \quad (8.23)$$

$$F_{\Phi_3} = \mu\Phi_3 + y\tilde{Q}_2Q_2 + \xi_3 \ . \quad (8.24)$$

Putting $\xi_2 = 0$ we see that besides the quartic term in the superpotential, we also get the mass terms

$$W \supset \frac{\xi_1}{\mu} Q_1 \tilde{Q}_1 + \frac{\xi_2}{\mu} Q_2 \tilde{Q}_2 \,. \tag{8.25}$$

In the geometry, the values of $\xi_1, \xi_2$ are exactly the sizes of the non-normalisable cycles in the geometry. For deformations of the $A_3$ singularity with quadratic superpotential for $\Phi$ we have three non-normalisable deformations, two of them being $\xi_1$ and $\xi_2$.

The solutions of the $F$-term equations imply that the bifundamentals become massive with masses $\xi_1/\mu$, $\xi_2/\mu$. In terms of the geometry we have three 2-cycles which we denote from left to right by $A$, $B$, $C$. There are $N_f - N_c$ D5 branes wrapped on the $A$ cycle, $N_c$ wrapped on the $B$ cycle and $N_c$ wrapped on the $C$ cycle. As above, the displacement of the $A$ cycle with respect to the $B$ cycle in their common normal bundle direction is equal to the size of the non-normalisable deformation $\xi_1$ and the displacement of the $C$ cycle with respect to the $B$ cycle in their common normal bundle direction is equal to the size of the non-normalisable deformation $\xi_2$.

We can now perform the Seiberg duality as a flop in the geometry or as a Weyl reflection in the $A_3$ algebra. We again interchange the $A$ and $C$ cycle, the change in cycles is

$$A \to A' = A + B, B' \to -B, C \to C' = C + B \tag{8.26}$$

In the Seiberg dual, there are $N_f - N_c$ D5 branes wrapped on $A'$ cycle, $N_c$ anti D5 branes on the $B'$ cycle and $N_c$ D5 branes on the $C'$ cycle. There are tachyons between the D5 branes and anti D5 branes. There should be a tachyon condensation in order to obtain a supersymmetric configuration. If $\xi_1 > \xi_2$, the condensation is between the branes wrapped on the $B'$, $C'$ cycles. If $\xi_1 < \xi_2$, the condensation is between the branes wrapped on the $A'$, $B'$ cycles.

If we consider the case of [71] where $\xi_1 > \xi_2$, the condensation appears between the $N_c$ anti D5 branes on the $B'$ cycle and $N_c$ D5 branes on the $C'$ cycle. The result is that the $N_f - N_c$ D5 branes wrapped on the $A'$ cycle remain unchanged and the other D5 branes are wrapped on a cycle which is bent in the $x'$ direction, where $x'$ is obtained from $x$ by a rotation of $\theta$. The dependence of $x'$ on $y$ is again monotonic due to the fact that the right most NS brane is at infinity.

# Chapter 9

# SQCD Vacua and Geometrical Engineering

This chapter is based on the publication [30] done in collaboration with Radu Tatar. In the present chapter we are going to give an alternative IIB geometrical engineering picture together with its T-dual IIA brane configuration with only NS branes and D4 branes for the field theory and IIA brane configuration of [17,23]. We start with the underlying $\mathcal{N}=2$ theories and obtain the $\mathcal{N}=1$ theories as deformations of $\mathcal{N}=2$ theories.

The ISS model [15] is obtained by starting with an $\mathcal{N}=2$, $SU(N_f) \times SU(N_c)$ theory, supersymmetry is broken with zero vevs for the electric quarks:

$$\mathcal{N}=2,\ SU(N_f) \times SU(N_c) \rightarrow \mathcal{N}=1,\ SU(N_f) \times SU(N_c)\ . \tag{9.1}$$

However, the breaking of supersymmetry in [17,23] allows vevs for the electric quarks:

$$\mathcal{N}=2,\ SU(N_f) \times SU(N_c) \rightarrow \mathcal{N}=1,\ SU(N_f-k) \times SU(N_c-k) \times SU(k)\ , \tag{9.2}$$

with a general integer value of $0 < k < \max\{N_f, N_c\}$. Looking at the brane configurations of the theories we can easily see why this is so. Labelling the masses of the fields in the adjoint representations of the gauge groups $SU(N_f)$

and $SU(N_c)$ in an obvious way, we see that if the masses of the adjoint fields are infinite, then the vacuum expectation values for the $k$ bifundamental fields given by

$$< Q\tilde{Q} >= \frac{\mu}{\frac{1}{m_{N_f}} + \frac{1}{m_{N_c}}} , \tag{9.3}$$

are infinite and the $SU(N_f - k) \times SU(N_c - k)$ factor is completely decoupled from the $SU(k)$ factor.

Seiberg duality can be viewed as a flop in the geometry and the dynamics of the branes is related to tachyon condensation between D5-branes and anti D5-branes [29]. In the present discussion the tachyon condensation explains the ranks of the groups in the magnetic theory, it also determines a reorientation of the cycles such that they remain holomorphic. This is related to the possibility that the cycles can slide along their normal bundles [1]. In the geometry of the metastable solutions of [17, 23], there are some extra D5-branes wrapped on certain 2-cycles. In the Seiberg dual picture they become D5-branes wrapped on non-holomorphic cycles. The non-holomorphic cycles can be deformed into holomorphic cycles in some limits. The deformation of the non-supersymmetric geometry in order to obtain a supersymmetric configuration is related to the lifetime of the metastable vacua.

## 9.1 The Field Theory

Start with an $\mathcal{N} = 2$, $SU(N_f) \times SU(N_c)$ model. This has two fields $\Phi, \tilde{\Phi}$ in the adjoint representation of the $SU(N_f)$ and $SU(N_c)$ groups and $N_f$ flavours $f$, $\tilde{f}$ in the fundamental and anti-fundamental representations of the gauge group respectively. Add the superpotential

$$W = \frac{1}{2}\tilde{\mu}\tilde{\Phi}^2 + \frac{1}{2}\mu\Phi^2 + \tilde{f}(\lambda\Phi + \tilde{\lambda}\tilde{\Phi})f + \xi\Phi + \tilde{\xi}\tilde{\Phi} , \tag{9.4}$$

[1]This was not allowed for the geometries of [15] where the cycles were fixed.

which breaks the supersymmetry $\mathcal{N} = 2 \to \mathcal{N} = 1$. The $F$-term equations of the superpotential are

$$
\begin{aligned}
F_{\tilde{f}} &= \lambda\Phi f + \tilde{\lambda} f \tilde{\Phi} = 0 \ , \\
F_f &= \lambda \tilde{f} \Phi + \tilde{\lambda} \Phi \tilde{f} = 0 \ , \\
F_\Phi &= \mu\Phi + \lambda f \tilde{f} + \xi = 0 \ , \\
F_{\tilde{\Phi}} &= \tilde{\mu}\tilde{\Phi} + \tilde{\lambda} \tilde{f} f + \tilde{\xi} = 0 \ .
\end{aligned} \tag{9.5}
$$

Consider the limit of infinite mass for the adjoint field of $SU(N_c)$ :$\tilde{\Phi}$. Integrating out this massive field via its equation of motion we find the superpotential

$$
W = \frac{1}{2}\mu\Phi^2 + \lambda \tilde{f} \Phi f + \xi\Phi \ . \tag{9.6}
$$

(9.6) can be identified with the electric superpotential of [17] via the relations

$$
\Phi \leftrightarrow N, \ \ f \leftrightarrow Q, \ \ \tilde{f} \leftrightarrow \tilde{Q}, \ \ \mu \leftrightarrow -\alpha_e, \ \ \xi \leftrightarrow m_e, \ \ \lambda \leftrightarrow -\frac{1}{\Lambda} \ , \tag{9.7}
$$

or the magnetic superpotential via

$$
\Phi \leftrightarrow M, \ \ f \leftrightarrow q, \ \ \tilde{f} \leftrightarrow \tilde{q}, \ \ \mu \leftrightarrow \alpha, \ \ \xi \leftrightarrow -m, \ \ \lambda \leftrightarrow \frac{1}{\Lambda} \ . \tag{9.8}
$$

Consider the electric theory, taking the limits

$$
\mu \to \infty, \ \ \frac{\lambda^2}{\mu} \to 0, \frac{\xi\lambda}{\mu} \to m \ , \tag{9.9}
$$

we recover the electric theory of the ISS model where $m$ is the mass of the electric quarks.

Now, the $\mathcal{N} = 2$, $SU(N_f) \times SU(N_c)$ theory is broken to an $\mathcal{N} = 1$, $SU(N_f) \times SU(N_c)$ theory with light quarks if the quantity $\frac{\xi\lambda}{\mu}$ is small. If however the term $\frac{\lambda^2}{\mu}$ is finite, we find that the $\mathcal{N} = 1$ superpotential is

$$
W = \frac{\lambda^2}{2\mu}(\tilde{f} f)^2 + \frac{\xi\lambda}{\mu}\tilde{f} f \ . \tag{9.10}
$$

We use the following form for the electric and magnetic superpotentials:

$$\begin{aligned} W_{ele} &= \tfrac{\lambda_e^2}{2\mu_e}(\tilde{Q}Q)^2 + \tfrac{\xi_e\lambda_e}{\mu_e}\tilde{Q}Q \ , \\ W_{mag} &= \tfrac{\lambda_m^2}{2\mu_m}(\tilde{Q}Q)^2 + \tfrac{\xi_m\lambda_m}{\mu_m}\tilde{Q}Q \ . \end{aligned} \tag{9.11}$$

A general supersymmetric solution of the $F$-term equations (9.5) with a finite mass for the adjoint field of the $SU(N_f)$ group and an infinite mass for the adjoint field of the $SU(N_c)$ is given by

$$\langle \tilde{f} f \rangle = \begin{pmatrix} -\frac{\lambda}{\xi}\mathbb{I}_k & 0 \\ 0 & 0 \end{pmatrix} . \tag{9.12}$$

This determines a breaking of the $SU(N_f) \times SU(N_c)$ group to $SU(N_f - k) \times SU(N_c - k) \times SU(k)$.

In terms of the coefficients of (9.11), the relations between the electric and magnetic variables are

$$\lambda_m = -\lambda_e, \quad \mu_m = -\frac{\lambda_e^2}{\mu_e}, \quad \xi_m = -\frac{\xi_e \lambda_e}{\mu_e} \ . \tag{9.13}$$

## 9.2 The Corresponding Geometry

Begin with a resolved $\mathcal{N} = 2$, $A_3$ singularity. Each of its three $P^1$ cycles has normal bundle:

$$X' = X, \quad Y' = YZ^2, \quad Z' = 1/Z \ . \tag{9.14}$$

This geometry is T-dual to four parallel $NS5$ branes.

We can deform the $\mathcal{N} = 2$, $A_3$ singularity into a collection of three resolved conifold singularities, each one looking like

$$X' = XZ, \quad Y' = YZ, \quad Z' = 1/Z \ , \tag{9.15}$$

breaking the supersymmetry to $\mathcal{N} = 1$. The T-dual is a set of two parallel NS branes and two orthogonal NS branes. In the present work, we consider a deformation to the $\mathcal{N}=2,\ A_3$ singularity similar to that discussed in [7]:

$$X' = X_r Z, \quad Y' = YZ, \quad Z' = 1/Z \ , \tag{9.16}$$

where the rotated direction $X_r$ is defined by

$$X_r = X - \frac{1}{m_{adj}} YZ \ , \tag{9.17}$$

and $m_{adj}$ is the mass of the field in the adjoint representation. This direction is chosen such that in the blow-down map

$$x \ = \ X \ = X'Z',\ y = ZY \ = \ Y',\ u \ = \ ZX \ = \ X'\ ,\ v \ = \ Y \ = \ Z'X' \tag{9.18}$$

we find the deformation of the singular conifold

$$u\ v \ - \ y\ (x \ - \ \frac{1}{m_{\mathrm{adj}}}\ y) = 0. \tag{9.19}$$

In the limit of infinite $m_{adj}$ we recover the usual conifold geometry while in the limit of vanishing $m_{adj}$ we can rescale $u$ and $v$ so we recover the case of (9.14).

For the field theory of Section 9.1 we have three adjoint field masses: one infinite, one finite and one zero. By rescaling $u$ and $v$, the geometry (9.17) is T-dual to a system of four NS branes in the directions $y$, $y$ cos $\theta - x$ sin $\theta$, where $\tan\theta = m_{adj}$. In the electric theory we identify the directions $y$ with $v = x^4 + ix^5$ (the direction of unrotated NS branes) and $x$ with $w = x^8 + ix^9$ (the direction of NS branes rotated at 90 degrees angle), and denote

$$v_\theta \ = \ w \sin\theta \ - \ v \cos\theta \ . \tag{9.20}$$

The electric theory contains from left to right an NS brane in the direction $v_\theta$, an NS branes in the direction $v$ and two NS$'$-branes in the direction $w$. We recover the field theory of the $\mathcal{N}=1, SU(N_f)\times SU(N_c)$ model by putting $N_f$ flavour D4-branes between the $v_\theta$ and $v$ NS-branes and $N_c$ colour D4-branes between the $v$ and $w$ NS-branes[2].

In the limit $\theta = \frac{\pi}{2}$ the $v_\theta$ and $w$ NS-branes are parallel in the direction $w$ and we obtain the ISS model. This was considered in [28] and Chapter 7. The displacement of these parallel NS-branes in the $v$ direction is identified with the mass of the quarks in the electric theory. In the geometrical engineering, the intervals between NS-branes become $P^1$ cycles. We label the cycles corresponding to the interval between the $v_\theta$ and $v$ NS branes by $C_1$, the interval between the $v$ and $w$ NS branes by $C_2$, and the interval between the $v_\theta$ and $w$ NS branes by $C_3$.

The works [12, 13, 62] considered Seiberg duality as a toric duality on the geometry. When performing the Seiberg duality the toric duality implies shrinking down the cycle $C_1$ and blowing up a cycle $C_4$.

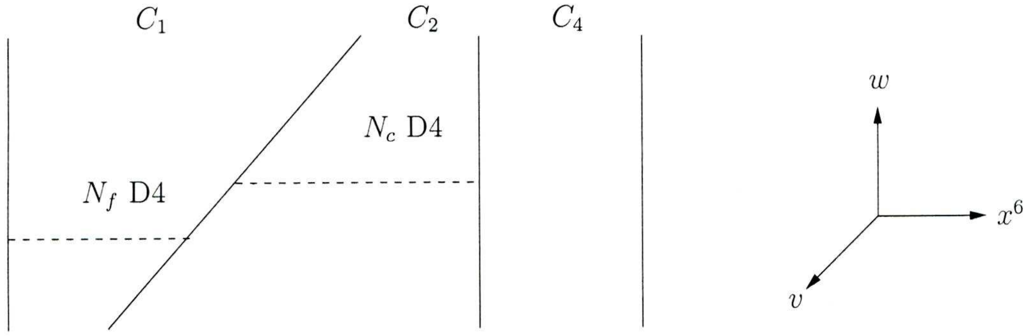


Figure 9.1: Electric configuration of branes with $N_f$ Massive Quarks

In the geometry of the ISS model after a toric duality the magnetic theory has a tachyon instability between the $N_c$ anti D5-branes wrapped on the $C_2$ cycle, and the $N_f$ D5-branes wrapped on the $C_5 = C_2 + C_4$ cycle. Since the cycles are fixed by the rigidity of their normal bundles, the only way we can have tachyon condensation is to deform the geometry in a non-holomorphic way [3]. The end result [28] is $N_f - N_c$ D5-branes wrapped on the $C_5 = C_2 + C_4$

[2]We choose this configuration because we do not want an extra adjoint in the gauge group (the reason for taking the colour D4 branes between orthogonal NS branes) and want a finite mass adjoint field for the flavour group (the reason for taking the flavour D4 branes between non-orthogonal NS branes).

[3]In the brane configuration picture, the main obstacle against a redistribution of D-

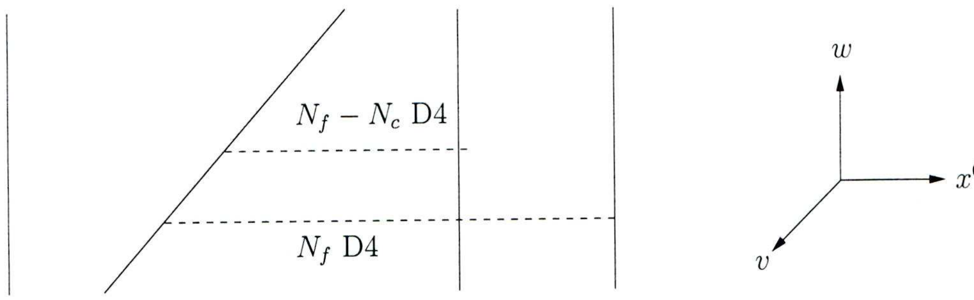


Figure 9.2: Magnetic configuration of branes with $N_f$ Massive Quarks

cycle and $N_c$ D5 branes wrapped on a non-holomorphic cycle.

The flop in the geometry exchanges the directions of the normal bundles $v \leftrightarrow w$. For consistency and considering the definition of $X_r$, we see that we need to take $m_{adj} \leftrightarrow \frac{1}{m_{adj}}$. In terms of brane configurations this implies that the relation between the rotation angles in the electric and magnetic pictures is

$$\theta_e = \pi/2 - \theta_m \ . \tag{9.21}$$

There are terms in the superpotential that are dependent on the rotation angles. We briefly review the appearance of the rotation angles in the expressions for the adjoint masses and Yukawa couplings. In the brane configuration of the electric theory the Yukawa coupling of the $\mathcal{N} = 2, SU(N_c)$ theory with $N_f$ fundamental flavours $Q$, $\tilde{Q}$ and adjoint field $\Phi$ is

$$Q\Phi\tilde{Q} \ . \tag{9.22}$$

This corresponds to $N_f$ D4-branes stretched between two parallel NS-branes extended in the $v$ direction. A rotation of one of the NS-branes by an angle

branes on holomorphic cycles is due to the fact that the ends of D4-branes on the $v$ NS brane can touch each other, whereas the other ends of the D4-branes cannot since they are stuck on parallel NS branes. In order to remove this problem, we need to make the two components of the normal bundle (or the NS branes) non-parallel *i.e.* by considering a field in the adjoint representation of the flavour group with finite mass.

$\theta$ into $v_\theta = -v\cos\theta + w\sin\theta$ changes the Yukawa coupling to

$$\lambda_e(\theta) Q\Phi\tilde{Q} \ , \tag{9.23}$$

where

$$\lambda_e(\theta) = -\cos\theta \ . \tag{9.24}$$

In the brane configuration of the magnetic theory the Yukawa coupling for the $\mathcal{N}=2$, $SU(N_f - N_c)$ theory with $N_f$ fundamental flavours $q$ is

$$q\Phi\tilde{q} \ . \tag{9.25}$$

This corresponds to $N_f$ D4-branes stretched between two parallel NS-branes extended in the $w$ direction.

The adjoint field of the gauge group $SU(N)$ of the $\mathcal{N}=2$ theory acquires a mass of $\tan\theta$ when two parallel NS-branes with $N$ D4-branes stretched between them are rotated with respect to one another by an angle $\theta$. Thus, breaking supersymmetry to $\mathcal{N}=1$.

We can set the coefficient in (9.25) to one by starting with orthogonal NS branes in the electric theory and keep track of the exchange $v \leftrightarrow w$ when considering the flop. The situation becomes more complicated if some NS-branes are rotated in the electric picture. In this case we need to keep track of the different normal bundles to various $P^1$ cycles wrapped with D5-branes. We need to make the following identifications:

1. Unrotated directions $v$ and $w$ are parts of the normal bundle to the $P^1$ and they are exchanged during the flop.

2. The angles of rotation in the electric theory are with respect to the axis $v$. After the flop, the $v$ and $w$ axes are exchanged and the rotation angle $\theta$ with respect to the axis $v$ becomes a rotation angle $\theta$ with respect to the axis $w$. This is equivalent to keeping the angle of rotation with respect to the $v$ axis before and after the flop and changing the rotation angle to $\pi/2 - \theta$ after the flop.

To obtain the geometries of the models of [17,23] we consider the following deformation of the $\mathcal{N}=2,\ A_1$ singularity to

$$X' = XZ, \quad Y' = Y_r Z, \quad Z' = 1/Z \ , \tag{9.26}$$

where $Y_r$ is defined by

$$Y_r = Y - \frac{1}{m_{adj}} XZ \ . \tag{9.27}$$

In the blow-down map

$$x = X \ = X'Z', \quad y = ZY = Y, \quad u \ = ZX = X', \quad v = Y \ = Z'X' \ , \tag{9.28}$$

we find the deformation of the singular conifold

$$uv - x(y - \frac{1}{m_{adj}} x) = 0 \ . \tag{9.29}$$

As before, we identify $x$ with $w$ and $y$ with $v$. The T-dual picture gives two NS-branes, one in the direction $w$ and the other in the direction

$$v_{\theta'} = v \ \sin\theta' - w\cos\theta' \ , \tag{9.30}$$

where $\theta' = \pi/2 - \theta$.

We need a different sign for the masses of the $\mathcal{N}=2$ adjoint fields in the electric and magnetic theories. Thus, we take $\theta_m = -\theta_e$ such that

$$\tan\theta_m = -\tan\theta_e \ , \tag{9.31}$$

which then gives

$$\cos\theta'_m = \cos(\pi/2 - \theta_m) = \sin\theta_m = -\sin\theta_e = -\cos\theta'_e \ . \tag{9.32}$$

Hence, if in the electric theory the Yukawa coupling is given by (9.23) and the mass of the adjoint by $-\tan\theta$, then the Yukawa coupling of the magnetic theory is proportional to[4]

$$\lambda_m(\theta) = \cos\theta' \ , \tag{9.33}$$

[4]If we keep track of dimensions the Yukawa couplings should contain a $\frac{1}{\Lambda}$ factor.

*i.e.* the sign of the Yukawa couplings and the mass of the adjoint fields have different signs in the magnetic and electric theories. We choose the convention that the magnetic terms are positive and the electric terms are negative.

Now we wish to consider the effect of a Seiberg duality on the geometry of the Giveon-Kutasov model. To get the Giveon-Kutasov model we again deform the resolved $\mathcal{N}=2$, $A_3$ singularity but this time by (9.26). We again have three adjoint masses: one finite, one infinite and one zero. The T-dual model has $NS$-branes in the directions, from left to right, $v_\theta$, $v$ and two in $w$. We put $N_c$ colour D4-branes between the $v$ and $w$ NS-branes and $N_f$ flavour D4-branes between the $v_\theta$ and $v$ NS-branes.

The Seiberg dual picture contains, from left to right, $NS$-branes in the directions $w$, $v$, $w$ and $v_\theta$. There are $N_c$ anti D4-branes between the $v$ and $w$ $NS$-branes and $N_f$ D4-branes between the $v$ and $v_\theta$ $NS$-branes. In the magnetic picture, the cycle wrapped by the $N_f$ D5-branes can slide. Tachyon annihilation begins on the $v$ line and propagates to the right. If the $N_f$ D5 branes were located at $(v,w)=(v_2,0)$ in their normal bundle and wrapped on the cycle $C_5=C_2+C_4$, the part lying on the $C_2$ cycle annihilates the $N_c$ anti D5-branes to give $N_f-N_c$ D5-branes wrapped on the $C_2$ cycle. While the other part is now on a $C_4$ cycle touching its normal bundle at a point $(0,-v_2\cot\theta)$. This is the dual $SU(N_f-N_c)$ theory with $N_f$ fundamental flavours.

### 9.2.1 Complex Deformations and Rearrangement of Cycles

As discussed in detail in [29] and Chapter 8, deformations of $\mathcal{N}=2$ geometries give rise to $\mathcal{N}=1$ geometries with normalisable and non-normalisable deformations. Normalisable deformations when translated into field theory effects can go through a geometric transition to study strong coupling effects. Non-normalisable deformations correspond to masses or vevs for the bifundamental $\mathcal{N}=2$ quarks [6].

By starting with a resolved $A_n$ singularity and adding $N_i$ D5 branes on each of the $i$ cycles we get the gauge group

$$\prod_{i=1}^{n} SU(N_i). \tag{9.34}$$

Adding a superpotential with quadratic terms for the adjoint fields, gives an $\mathcal{N}=1$ theory with [6]

$$n(n+1)/2 \qquad \text{normalisable deformations ,} \tag{9.35}$$

and

$$n(n-1)/2 \qquad \text{non-normalisable deformations .} \tag{9.36}$$

For the case of the deformation of $A_3$, there are six normalisable and three non-normalisable deformations. In the geometry of the ISS model, the mismatch of non-normalisable deformations in the electric and magnetic pictures can be used as an explanation why the ISS solution cannot be constructed with D5-branes wrapped on holomorphic cycles [29]. In the electric picture there are two masses for the adjoint fields whereas in the magnetic picture there is only mass for one adjoint field. Tachyon condensation in the magnetic theory corresponds to non-holomorphic deformations of the cycles.

For the case of the geometry of the Giveon-Kutasov model, there are two masses for the adjoint fields in both the electric and magnetic pictures and no mismatch of non-normalisable deformations. The non-normalisable deformations are visible in both electric and magnetic pictures as follows:

- In the electric picture the non-normalisable deformations correspond to masses for the light quarks. In terms of brane configurations, the non-normalisable deformations correspond to displacing the $N_f$ flavour D4-branes from the point $(v, w) = (0, 0)$ to a point $(v_Q, 0)$, where $v_Q \neq 0$.

- In the magnetic picture the non-normalisable deformations correspond to vevs for the adjoint field of the flavour group. In terms of brane configurations this corresponds to displacing the $N_f$ flavour D4-branes from the point $(v, w) = (0, 0)$ to the point $(0, w_{adj})$ where $w_{adj} \neq 0$.

- The tachyon condensation corresponds to exchanging the two types of non-normalisable deformations. The displacement in the $v$ direction between the flavour and colour D4-branes is mapped into a displacement of the D4-branes in the $w$ direction.

- From the geometry, we see that by starting with an electric displacement to $(v, 0)$, considering the flop and then the tachyon condensation by sliding the $N_f$ branes along the $v_\theta$ direction, we get a displacement to $(0, v \cot\theta_m)$. Recalling that $\theta_m = -\theta_e$, it results that the displacement is to $(0, -v\cot\theta_e)$.

### 9.2.2 $\mathcal{N}=2,\ SU(N_f)\times SU(N_c) \to \mathcal{N}=1,\ SU(N_f-k)\times SU(N_c-k)\times SU(k)$

The relation between the masses of the adjoints and the vev for the electric quarks is

$$< Q\tilde{Q} >= \frac{\mu}{\frac{1}{m_{N_f}}+\frac{1}{m_{N_c}}} \ . \tag{9.37}$$

If $m_{N_f}$ is finite, there are extra solutions [17, 23]. In terms of IIA brane configurations the extra solutions are obtained by displacing $k$ of the $N_f$ flavour D4-branes in the $w$ direction [5]. In terms of the geometrical picture, the extra wrapped D5-branes appear because for a deformed $A_2$ singularity with a quadratic superpotential there are three normalisable deformations corresponding to $P^1$ cycles. We can wrap $N_f-k$ D5-branes, $N_c-k$ D5-branes and $k$ D5-branes on distinct $P^1$ cycles. The colour $P^1$ cycle is wrapped by $N_c-k$ D5-branes and the flavour $P^1$ cycles by $N_f-k$ and $k$ D5-branes.

As discussed in [4–6], for an $A_2$ singularity deformed by the superpotential

$$W_1(v)+W_2(v), \quad v = \text{ vev of } \Phi_1 \text{ or } \Phi_2 \ , \tag{9.38}$$

the position of the D4-branes when one of the NS branes is unrotated is given

[5]From the $\mathcal{N}=2$ Yukawa coupling, the displacement along the $v$ direction is always associated to the mass of the electric quarks.

by

$$W_1'(v) = 0 \quad W_2'(v) = 0 \quad \text{and} \quad W_1'(v) + W_2'(v) = 0\ . \tag{9.39}$$

It is more convenient to consider that the unrotated NS-brane is the $v_\theta$ direction such that the $v$ NS-brane is now rotated by an angle $-\theta$ and the $w$ NS-brane by $\pi/2 - \theta$. In this case (9.39) now becomes

$$W_1'(v_\theta) = 0 \quad W_2'(v_\theta) = 0 \quad \text{and} \quad W_1'(v_\theta) + W_2'(v_\theta) = 0\ . \tag{9.40}$$

The D4-branes located at $W_1'(v_\theta) = 0$ stretch between the $v_\theta$ NS-brane and the $v$ NS-brane, the D4-branes located at $W_2'(v_\theta) = 0$ are stretched between the $v$ NS-brane and the $w$ NS$'$-brane and the D4-branes located at $W_1'(v_\theta) + W_2'(v_\theta) = 0$ are stretched between the $v_\theta$ NS brane and the $w$ NS$'$ brane.

We now consider a Seiberg duality which is a flop of the colour $P^1$ cycle. The normal bundles to the other $P^1$ cycles are also changed by the flop.

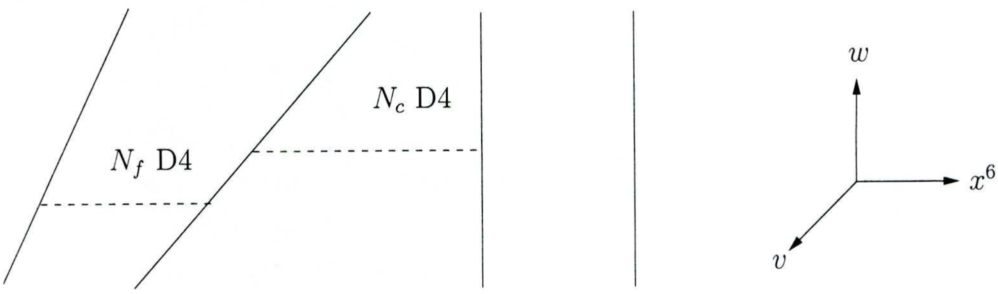


Figure 9.3: $k = 0$ Electric configuration with $v_{\theta'}$ NS brane

The reference NS-brane is now in the $v_{\theta'}$ direction and the positions of the D4-branes are given by

$$W_1'(v_{\theta'}) = 0 \quad W_2'(v_{\theta'}) = 0 \quad \text{and} \quad W_1'(v_{\theta'}) + W_2'(v_{\theta'}) = 0\ . \tag{9.41}$$

The $v$ NS-brane is now in the $w$ direction and the $v_{\theta'}$ NS-brane is now in the $v$ direction. The D4-branes located at $W_1'(v_{\theta'}) = 0$ are between the $v_{\theta'}$ NS-brane and the $w$ NS-brane, the D4-branes located at $W_2'(v_{\theta'}) = 0$ are between the $w$ NS-brane and the $v$ NS$'$-brane, and the D4-branes located at

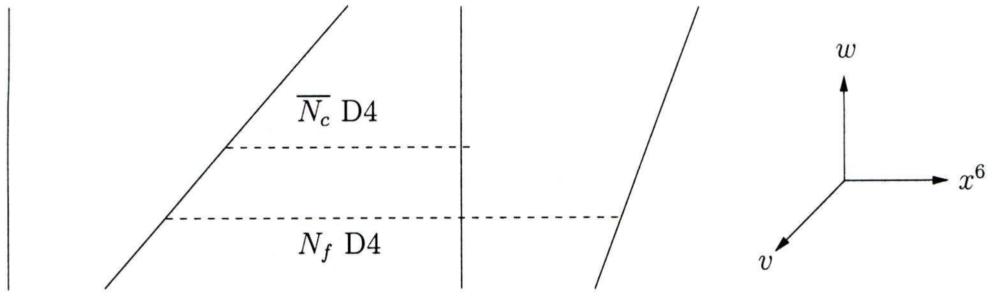


Figure 9.4: $k = 0$ Magnetic before tachyon condensation

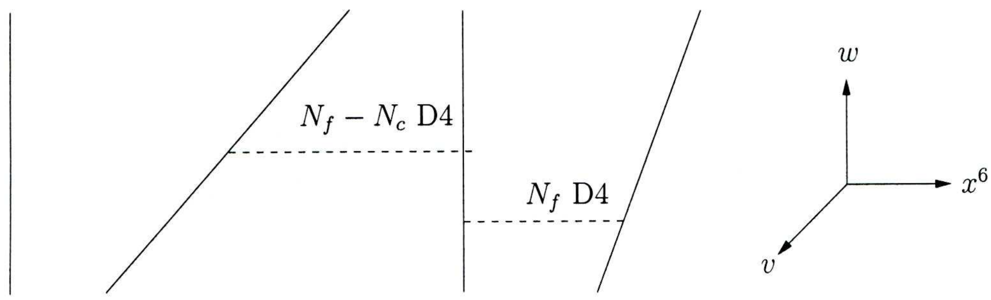


Figure 9.5: $k = 0$ Magnetic after tachyon condensation

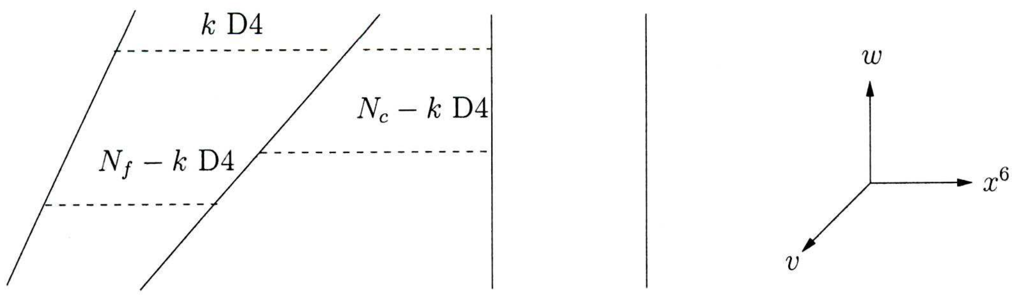


Figure 9.6: $k \neq 0$ Electric configuration

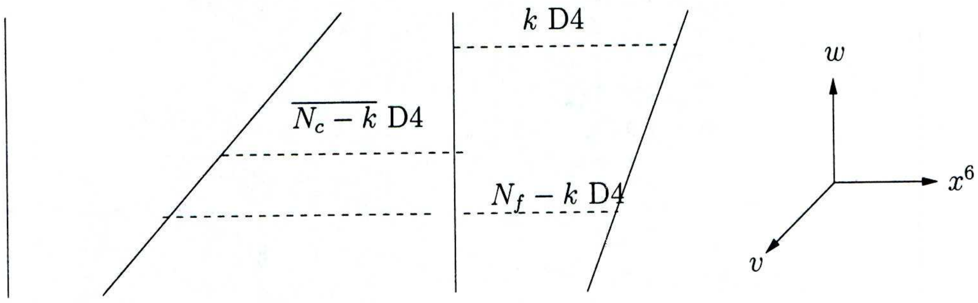


Figure 9.7: $k \neq 0$ Magnetic before tachyon condensation

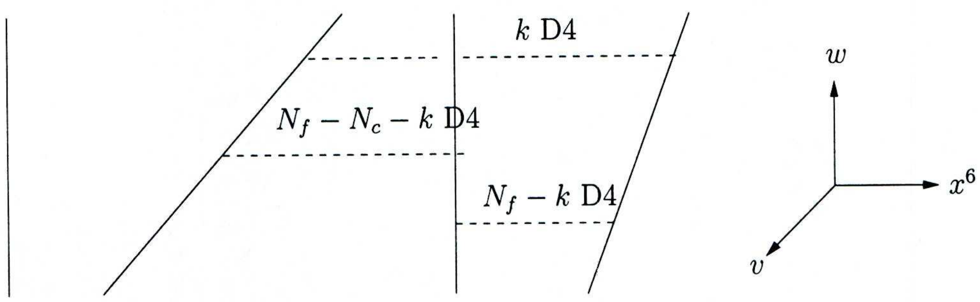


Figure 9.8: $k \neq 0$ Magnetic after tachyon condensation

$W_1'(v_{\theta'}) + W_2'(v_{\theta'}) = 0$ are between the $v_{\theta'}$ NS-brane and the $v$ NS$'$ brane.

When we consider the direct mapping of the cycles and the D4-branes during the flop there appears to be a problem. The D4-branes in the magnetic theory appear to be extended between the same NS-branes as in the electric theory. This would make sense only if the directions $v_\theta$ and $v_{\theta'}$ were identical which they are not.

We now discuss the change in the cycles wrapped by the D5 branes due to the flop. By denoting the cycles in the same manner as in Section 9.2, we have the following wrapped branes:

- The electric theory has $N_f - k$ D5-branes wrapping the $C_1$ cycle, $N_c - k$ D5-branes wrapping the $C_2$ cycle and $k$ D5-branes wrapping the $C_3$ cycle. The $C_1$ cycle has normal bundle $(v_\theta, v)$, the $C_2$ cycle has normal bundle $(v, w)$ and the $C_3$ cycle has normal bundle $(v_\theta, w)$.

- The magnetic theory obtained after the flop has $N_c - k$ anti D5-branes wrapping the $C_2$ cycle, $k$ D5-branes wrapping the $C_4$ cycle and $N_f - k$ D5-branes wrapping the $C_5$ cycle. The $C_4$ cycle has normal bundle $(w, v_{\theta'})$, the $C_2$ cycle has normal bundle $(v, w)$ *i.e.* it's normal bundle is unchanged, and the $C_5$ cycle has normal bundle $(v, v_{\theta'})$.

After the flop the cycle with wrapped colour branes changes orientation and the wrapped $N_c - k$ D5-branes become $N_c - k$ wrapped anti D5-branes. There are two types of tachyons between the D5-branes and anti D5-branes:

1. Between the $N_c - k$ anti D5-branes and the $N_f - k$ D5-branes.

2. Between the $N_c - k$ anti D5-branes and the $k$ D5-branes.

The first tachyon condensation is between the $N_f - k$ D4-branes and $N_c - k$ anti-D4 branes ending on the NS-brane at the same side. This is because of the results of [72]: when D4 and anti D4-branes end on the opposite sides of an NS-branes, they repel each other. Their argument uses the fact that the end of the D4 branes are charged 3-branes in the NS world volume. If we dimensionally reduce over these 3 dimensions, the ends are vortices living on the reduced NS world volume. We can apply this argument to the two types of pairs of D-branes anti D-branes. For the $k$ D4-branes and $N_c - k$ anti D4-branes ending on the NS-brane at opposite sides, the vortices carry the same charge under the gauge field and they repel. For the $N_f - k$ D4-branes and $N_c - k$ anti D4-branes ending on the NS-brane at the same side, the vortices carry different charges under the gauge field and they attract. The tachyon condensation changes the cycle wrapped by the $N_f - k$ D5-branes which will now have the normal bundle $w, v_{\theta'}$.

There are $N_f - k$ D5-branes wrapped on the $C_5$ cycle and $N_c - k$ anti D5-branes wrapped on the $C_2$ cycle. This configuration is unstable and there is a tunnelling process taking us to a stable configuration obtained by splitting the $C_5$ cycle into a $C_2$ cycle and a $C_4$ cycle such that there are $N_f - k$ D5-branes wrapped on both the $C_2$ and $C_4$ cycles. Annihilating the $N_f - k$ D5-branes wrapped on the $C_2$ cycle with the $N_c - k$ anti D5-branes wrapped on the $C_2$ cycle gives $N_f - k$ D5-branes wrapped on the $C_4$ cycle and $N_f - N_c$ D5-branes wrapped on the $C_2$ cycle.

We now examine what happens to the $k$ D5-branes wrapped on the $C_4$ cycle. As the $C_5$ cycle changes its normal bundle during the tachyon condensation, the $C_4$ cycle also changes its normal bundle. This can be understood in the following way: the separation between the cycles is related to an expectation value for the gauge singlet $\Phi$ or the gauge invariant combination $q\tilde{q}$. The vev for the gauge singlet $\Phi$ or for $q\tilde{q}$ are measured along different directions in the geometry. Before tachyon condensation the vev for $\Phi$ is measured along the $v$ direction and the vev for $q\tilde{q}$ is measured along the $w$ direction. After tachyon condensation the $N_f - k$ D5-branes wrapped on the $C_5$ cycle change into $N_f - k$ D5-branes wrapped on the $C_4$ and $C_2$ cycles. The vev for $\Phi$ is now measured along the $w$ direction meaning that the vev for $q\tilde{q}$ is now measured along the $v$ direction. This implies that the displacement of the $k$ D5-branes with respect to the colour branes should be made in the direction $v$. The only way to do this and still have the $k$ D5-branes wrapped on a holomorphic cycle is to wrap the $k$ D5-branes on the $C_5$ cycle. This process means that, in the first instance, the $k$ D5-branes are wrapped on a non-holomorphic cycle and the energy of the $k$ D5-branes is increased. We can use the the vortex argument to state that the $k$ D5-branes and the

$N_f - N_c$ colour D5-branes attract each other. The excess energy is lost when $k$ of the $N_f - N_c$ colour D5-branes combine with the $k$ D5-branes to give $k$ D5-branes wrapped on the $C_5$ cycle. We leave the detailed description of the combined process of tachyon condensation and cycle redistribution for future work.

The solutions obtained in this section are supersymmetric since (9.37) allows for a compatible deformation

$$C_4 \leftrightarrow C_5, \quad C_5 \leftrightarrow C_4 \tag{9.42}$$

In the next section we discuss the non-supersymmetric solutions of the Giveon-Kutasov model where there are flavour branes wrapped on a $C_5' \neq C_5$ cycle. In this case the $C_5'$ cycle is not exchangeable with the $C_4$ cycle and it remains non-holomorphic after tachyon condensation.

## 9.3 Metastable Vacua

Beside the supersymmetric vacua described above [17, 23] also considered a large set of metastable non-supersymmetric vacua. These appear when there is a further breaking of the flavour group:

$$SU(N_f - k) \to SU(N_f - k - n) \times SU(n) \ , \tag{9.43}$$

such that the masses of the $SU(N_f - k - n)$ flavours satisfy (9.37) whereas the masses of the $SU(n)$ flavours do not. This implies that not all the positions of the D4-branes in the brane picture are described by (9.41).

In the electric theory we have $N_f - k - n$ D5-branes wrapped on the $C_1$ cycle[6], $n$ D5-branes wrapped on a new $C_1'$ cycle, $N_c$ D5-branes wrapped on the $C_2$ cycle and $k$ D5-branes on the $C_3$ cycle. In this case, the $C_1$ and $C_1'$ cycles are displaced in the $v$ direction.

We now perform a Seiberg duality on this configuration. The flop on the colour cycle results in the following wrapped branes: $N_c - k$ anti D5-branes

[6]We use the same notation for the cycles as in previous sections.

wrapped on the $C_2$ cycle, $N_f - k - n$ D5-branes wrapped on the $C_5$ cycle , $n$ D5-branes wrapped on a $C_5'$ cycle and $k$ D5-branes on the $C_4$ cycle. The $C_5$ and $C_5'$ cycles are also displaced in the $v$ direction.

The validity of the analysis in [17] holds in the limit that the coefficients in (9.4) obey

$$\mu << \xi \,, \tag{9.44}$$

implying a very small magnetic angle of rotation. We can again use the results of [72] to argue that the $N_c - k$ anti D5-branes and the $k$ D5 branes repel each other and no tachyon condensation takes place between them. Tachyon condensation occurs first between the $N_c - k$ anti D5-branes and the $N_f - k - n$ D5-branes resulting in $N_f - N_c - n$ colour D5 branes.

The $N_f - k - n$ D5-branes are now wrapped on the $C_4$ cycle. We now consider the effect of the tachyon condensation on the $n$ and $k$ D5-branes. Because of the $C_4 \leftrightarrow C_5$ cycle interchange, the stack of $k$ D5-branes now becomes wrapped on the $C_5$ cycle due to (9.37). As the position of the $n$ D5-branes does not satisfy (9.37), they do not become wrapped on some $C_4'$ cycle. They wrap a non-holomorphic deformation of the $C_5$ cycle, denoted by $\tilde{C}$, making the configuration non-supersymmetric.

This non-supersymmetric configuration can decay into supersymmetric configurations in two ways:

1. When the $n$ D5-branes are close to the $k$ D5-branes, the switch of cycles $C_4 \leftrightarrow C_5$ is performed at the same time as $\tilde{C}$ changing into a holomorphic cycle. The result is $k+n$ D5-branes wrapped on the same $C_5$ cycle. The metastability of the vacua is related to the deformation of the non-holomorphic cycle $\tilde{C}$ into $C_5$. It would be interesting to show that the duration of this deformation is larger that the time necessary for the tachyon to condense, a crucial requirement for metastability.

   The final configuration is the same as the supersymmetric configuration obtained by $\mathcal{N}=2,\ SU(N_f)\times SU(N_c) \rightarrow \mathcal{N}=1,\ SU(N_f-k-n)\times SU(N_c-k-n)\times SU(k+n)$

   with the emphasis that there are some intermediate steps which should be covered in a longer time than the usual $\mathcal{N}=2,\ SU(N_f)\times SU(N_c) \rightarrow$

$\mathcal{N}=1$, $SU(N_f-k)\times SU(N_c-k)\times SU(k)$ breaking.

2. When the $n$ D5-branes are very far from the $k$ D5-branes and the colour D5 branes and are close to the $N_f-k-n$ D5-branes, the switch of cycles $C_4 \leftrightarrow C_5$ is again performed at the same time as the holomorphic change of the $\tilde{C}$ cycle. This results in $N_f-k-n+n = N_f-k$ D5-branes being wrapped on the $C_4$ cycle. The metastability is now related to the deformation of the non-holomorphic cycle, $\tilde{C}$, into the holomorphic cycle $C_4$. The final configuration is identical to the supersymmetric case where we start from an unbroken $SU(N_f-k)$ group in the electric theory. It would interesting to understand why the deformation of the $\tilde{C}$ cycle should take a much longer time than the tachyon condensations.

The above discussion is valid in the range $1\leq n\leq N_f-N_c-k$, when the number of colour D5 branes, $N_f-N_c-n-k$, is positive. When $n>N_f-N_c-k$ *i.e.* $N_f-N_c-n-k<0$, the first type of tachyon condensation described above does not occur, and the solution is more stable. All other cases considered in [17,23] can be expressed in terms of tachyon condensations between pairs of branes and anti branes and rearranging of cycles to holomorphic embeddings.

# Chapter 10

# Conclusions

The thesis focuses on geometrical pictures in IIB string theory of field theories with metastable vacua. We used T-dualities to convert IIA branes configurations with four and fivebranes to geometrically engineered configurations in IIB with wrapped fivebranes.

Chapter 7 studies the geometrical engineering of the metastable vacua proposed in [15,18]. The IIA brane configuration of [15] considered in [20,21] is translated into a system of D4 and NS branes and then a T-duality was performed to obtain a geometrical picture in IIB. The configuration of [18] with D5 branes and anti D5 branes separated by a potential barrier is translated into a IIA brane configuration by considering the effect of a T-duality, and the corresponding IIA brane configuration is lifted to M theory. In considering the ISS model w there are modifications to the complex structure for the resolution of $\mathcal{N} = 2$ singularities.

Non-normalisable complex deformations to describe stringy realisations of metastable vacua in $\mathcal{N} = 1$, $SU(N)$ gauge theories with $F > N$ massive fundamental flavours are considered in Chapter 8. Considering non-normalisable deformations leads to a modified toric duality. After a Seiberg duality we are left with a configuration containing branes and anti-branes and the corresponding tachyon condensation between pairs of wrapped D5 and anti D5 branes results in a mixing between the cycles in the geometry. We enlarge the class of metastable vacua to the case of branes-antibranes wrapped on cycles of deformed $A_n$ singularities.

Chapter 9 covers the geometrical engineering of the $\mathcal{N} = 1$ brane configuration of [23] whose field theory is considered in [17]. We performed a T-duality on the brane configurations to obtain geometrically engineered theories in IIB string theory. The field theories encoded by the geometries contain extra massive adjoint fields for the flavour group. A flop in the geometry gives a system of branes and antibranes and branes wrapped on non-holomorphic cycles. The various tachyon condensations between the branes and anti-branes give rise to a variety of supersymmetric and metastable non-supersymmetric vacua.

The decay from a theory with a non-holomorphic cycles to a theory with only holomorphic cycles is related to the lifetime of the metastable vacua. The duration of deformation should be larger than the time necessary for tachyon condensation. It is not yet understood why this should be the case.

In the IIA picture the lifetime of the metastable vacua is related to the change in length of the branes when going from the non-supersymmetric configuration to the supersymmetric configuration. In IIB the lifetime is related to a deformation of a non-holomorphic cycle to a holomorphic cycle. It is not yet sure how to estimate the lifetime of the vacua in a quantitative sense, unlike in the field theory and in [18], where they could make the metastable vacua parametrically long lived.

[27] translated the one loop corrections due to the Coleman-Weinberg potential in the field theory into a classical gravitational attraction between fourbranes and fivebranes in IIA brane configurations. However, quantum effects appearing in the field theory as non-perturbative corrections, are yet to be understood.

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